Partial fractions 1 : Answers

June 2004

1) a) $$\frac{14x^2 - 11x - 3}{(x+2)(2x-1)^2} = \frac{A}{(x+2)} + \frac{B}{(2x-1)} + \frac{C}{(2x-1)^2}$$

$$14x^2 - 11x - 3 = A(2x-1)^2 + B(x+2)(2x-1) + C(x+2)$$

$\underline{x = -2}$

$$56 + 22 - 3 = A(-5)^2 + 0B + 0C$$
$$75 = 25A$$
$$3 = A$$

$\underline{x = \tfrac{1}{2}}$

$$\frac{14}{4} - \frac{11}{2} - 3 = 0A + 0B + \frac{5}{2}C$$
$$\frac{7}{2} - \frac{11}{2} - \frac{6}{2} = \frac{5}{2}C$$
$$7 - 11 - 6 = 5C$$
$$-10 = 5C$$
$$-2 = C$$

$\underline{x = 0}$

$$-3 = A(-1)^2 + B(2)(-1) + C(2)$$
$$-3 = A - 2B + 2C$$
$$-3 = 3 - 2B - 4$$
$$2B = 2$$
$$B = 1$$

$$\therefore \frac{14x^2 - 11x - 3}{(x+2)(2x-1)^2} = \frac{3}{(x+2)} + \frac{1}{(2x-1)} - \frac{2}{(2x-1)^2}$$

b) $$\int \frac{14x^2 - 11x - 3}{(x+2)(2x-1)^2}\,dx = 3\int \frac{1}{x+2}\,dx + \int \frac{1}{2x-1}\,dx - 2\int (2x-1)^{-2}\,dx$$

$$= 3\ln|x+2| + \frac{1}{2}\ln|2x-1| - \frac{2(2x-1)^{-1}}{(-1)\times 2} + c$$

$$= 3\ln|x+2| + \frac{1}{2}\ln|2x-1| + \frac{1}{(2x-1)} + c$$

June 2003

$$\frac{3x^2+2x+1}{(1+2x)(1+x)^2} = \frac{A}{(1+2x)} + \frac{B}{(1+x)} + \frac{C}{(1+x)^2}$$

$$3x^2+2x+1 = A(1+x)^2 + B(1+2x)(1+x) + c(1+2x)$$

$\underline{x=-1}$

$$3-2+1 = 0A + 0B + c(-1)$$

$$C = -2$$

$\underline{x=-1/2}$

$$3\left(\frac{1}{4}\right) - 1 + 1 = A(1/2)^2 + 0B + 0C$$

$$\frac{3}{4} = \frac{A}{4}$$

$$3 = A$$

$\underline{x=0}$

$$1 = A(1) + B(1)(1) + c(1)$$

$$1 = 3 + B - 2$$

$$0 = B$$

$$\therefore \quad \frac{3x^2+2x+1}{(1+2x)(1+x)^2} = \frac{3}{(1+2x)} - \frac{2}{(1+x)^2}$$

$$\therefore \int \frac{3x^2+2x+1}{(1+2x)(1+x)^2}\,dx = 3\int \frac{1}{1+2x}\,dx - 2\int (1+x)^{-2}\,dx$$

$$= \frac{3}{2}\ln|1+2x| - \frac{2(1+x)^{-1}}{(-1)\times 1} + C$$

$$= \frac{3}{2}\ln|1+2x| + \frac{2}{(1+x)} + C$$

June 2002

$$f(x) = \frac{x^2+2x-1}{(2+x)^2(1+x)} = \frac{A}{(2+x)} + \frac{B}{(2+x)^2} + \frac{C}{(1+x)}$$

$$x^2+2x-1 = A(2+x)(1+x) + B(1+x) + C(2+x)^2$$

$\underline{x=-2}$

$$4-4-1 = 0A - B + 0C$$
$$-1 = -B$$
$$B = 1$$

$\underline{x=-1}$

$$1-2-1 = 0A + 0B + C$$
$$-2 = C$$

$\underline{x=0}$

$$-1 = A(2)(1) + B(1) + C(4)$$
$$-1 = 2A + B + 4C$$
$$-1 = 2A + 1 - 8$$
$$6 = 2A$$
$$3 = A$$

$$\therefore f(x) = \frac{3}{(2+x)} + \frac{1}{(2+x)^2} - \frac{2}{(1+x)}$$

$\underline{f'(x)}$

$$f(x) = 3(2+x)^{-1} + (2+x)^{-2} - 2(1+x)^{-1}$$

$$f'(x) = -3(2+x)^{-2}\times 1 - 2(2+x)^{-3}\times 1 + 2(1+x)^{-2}\times 1$$

$$= \frac{-3}{(2+x)^2} - \frac{2}{(2+x)^3} + \frac{2}{(1+x)^2}$$

$$f'(1) = \frac{-3}{3^2} - \frac{2}{3^3} + \frac{2}{2^2}$$

$$= -\frac{1}{9} - \frac{2}{27} + \frac{1}{2} = -\frac{6}{54} - \frac{4}{54} + \frac{27}{54}$$

$$= \frac{17}{54}$$

June 2001

a) $$\frac{1}{x^2(2x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x-1)}$$

$$1 = Ax(2x-1) + B(2x-1) + Cx^2$$

$\underline{x = 0}$

$$1 = 0A + B(-1) + 0C$$

$$-1 = B$$

$\underline{x = 1/2}$

$$1 = 0A + 0B + \frac{1}{4}C$$

$$4 = C$$

$\underline{x = 1}$

$$1 = A(1)(1) + B(1) + C(1)$$
$$1 = A - 1 + 4$$
$$-2 = A$$

$$\therefore \frac{1}{x^2(2x-1)} = \frac{-2}{x} - \frac{1}{x^2} + \frac{4}{(2x-1)}$$

Now $$\int \frac{1}{x^2(2x-1)}\,dx = -2\int \frac{1}{x}\,dx - \int x^{-2}\,dx + 4\int \frac{1}{2x-1}\,dx$$

$$= -2\ln|x| - \frac{x^{-1}}{-1} + \frac{4}{2}\ln|2x-1| + c$$

$$= -2\ln|x| + \frac{1}{x} + 2\ln|2x-1| + c$$

June 2005

$$\frac{8x^2+x-5}{(2x-1)^2(x+2)} = \frac{A}{(2x-1)} + \frac{B}{(2x-1)^2} + \frac{C}{(x+2)}$$

$$8x^2+x-5 = A(2x-1)(x+2) + B(x+2) + C(2x-1)^2$$

$\underline{x=-2}$

$$32-2-5 = 0A + 0B + C(-5)^2$$
$$25 = 25C$$
$$1 = C$$

$\underline{x = 1/2}$

$$8\left(\frac{1}{4}\right) + \frac{1}{2} - 5 = 0A + B\left(5/2\right) + 0C$$
$$-\frac{5}{2} = \frac{5B}{2}$$
$$-1 = B$$

$\underline{x=0}$

$$-5 = A(-1)(2) + B(2) + C(-1)^2$$
$$-5 = -2A + 2B + C$$
$$-5 = -2A - 2 + 1$$
$$2A = 4$$
$$A = 2$$

$$\therefore \frac{8x^2+x-5}{(2x-1)^2(x+2)} = \frac{2}{(2x-1)} - \frac{1}{(2x-1)^2} + \frac{1}{(x+2)}$$

Now $\displaystyle\int \frac{8x^2+x-5}{(2x-1)^2(x+2)}\,dx = 2\int\frac{1}{2x-1}\,dx - \int(2x-1)^{-2}\,dx + \int\frac{1}{(x+2)}\,dx$

$$= \frac{2}{2}\ln|2x-1| - \frac{(2x-1)^{-1}}{(-1)\times 2} + \ln|x+2|$$

$$= \ln|2x-1| + \frac{1}{2(2x-1)} + \ln|x+2|$$