Trigonometry 5 : Identity Proof : Answers

1) $\sin\theta + \cos\theta \equiv (\tan\theta + 1)\cos\theta$

RHS $(\tan\theta + 1)\cos\theta$

$= \left(\frac{\sin\theta}{\cos\theta} + 1\right)\cos\theta$

$= \sin\theta + \cos\theta$

$=$ LHS QED.

2) $\sin x = \frac{o}{h}$ $\quad \cos x = \frac{a}{h}$ $\quad$ Prove $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = \frac{o^2}{h^2}$ $\quad \cos^2 x = \frac{a^2}{h^2}$

$\therefore$ LHS $\sin^2 x + \cos^2 x$

$= \frac{o^2}{h^2} + \frac{a^2}{h^2}$ $\quad$ From Pythag $h^2 = o^2 + a^2$

$= \frac{o^2 + a^2}{h^2}$

$= \frac{h^2}{h^2}$

$= 1 =$ RHS QED.

3) $\sin^3\theta \equiv \tan\theta\cos\theta - \sin\theta\cos^2\theta$

RHS $\tan\theta\cos\theta - \sin\theta\cos^2\theta$

$= \frac{\sin\theta}{\cos\theta}\cos\theta - \sin\theta(1 - \sin^2\theta)$

$= \sin\theta - \sin\theta + \sin^3\theta$

$= \sin^3\theta$

$=$ LHS QED.

4) $\sin\theta = \frac{o}{h}$ $\quad \cos\theta = \frac{a}{h}$ $\quad \tan\theta = \frac{o}{a}$

Prove $\frac{\sin\theta}{\cos\theta} = \tan\theta$

LHS $\frac{\sin\theta}{\cos\theta}$

$= \frac{o/h}{a/h} = \frac{o}{h} \times \frac{h}{a}$

$= o/a$

$= \tan\theta =$ RHS QED.

5) $3 - 2\cos^2 x \equiv 2\sin^2 x + 1$

$$\begin{aligned} \text{LHS} \quad & 3 - 2\cos^2 x \\ &= 3 - 2(1 - \sin^2 x) \\ &= 3 - 2 + 2\sin^2 x \\ &= 1 + 2\sin^2 x \\ &= \text{RHS} \qquad \text{QED} \end{aligned}$$

6) $\cos\theta\,(3\cos^2\theta - \tan\theta) \equiv 3\cos\theta - 3\sin^2\theta\cos\theta - \sin\theta$

$$\begin{aligned} \text{LHS} &= \cos\theta\left[3(1-\sin^2\theta) - \frac{\sin\theta}{\cos\theta}\right] \\ &= \cos\theta\left[3 - 3\sin^2\theta - \frac{\sin\theta}{\cos\theta}\right] \\ &= 3\cos\theta - 3\sin^2\theta\cos\theta - \sin\theta \\ &= \text{RHS} \qquad \text{QED.} \end{aligned}$$

7) $\cos\theta(2\tan\theta - 3\sin^2\theta) \equiv 2\sin\theta - 3\cos\theta + 3\cos^3\theta$

$$\begin{aligned} \text{LHS} &= 2\cos\theta\tan\theta - 3\cos\theta\sin^2\theta \\ &= 2\cos\theta\frac{\sin\theta}{\cos\theta} - 3\cos\theta(1-\cos^2\theta) \\ &= 2\sin\theta - 3\cos\theta + 3\cos^3\theta \\ &= \text{RHS} \qquad \text{QED.} \end{aligned}$$

8) $1 - \tan^2 x \equiv \dfrac{\sin^2 x(\cos^2 x - 1) + \cos^4 x}{\cos^2 x}$

NOTICE $\div \cos^2 x$

$$\begin{aligned} \text{RHS} &= (\sin^2 x\cos^2 x - \sin^2 x + \cos^4 x) / \cos^2 x \\ &= (\sin^2 x\cos^2 x - \sin^2 x + \cos^2 x\cos^2 x) / \cos^2 x \\ &= [\cos^2 x(\sin^2 x + \cos^2 x) - \sin^2 x] / \cos^2 x \\ &= \\ &= \frac{\cos^2 x(1) - \sin^2 x}{\cos^2 x} \\ &= \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = 1 - \frac{\sin^2 x}{\cos^2 x} \\ &= 1 - \tan^2 x = \text{LHS} \end{aligned}$$

I forgot to put it in at first

QED.