

Completing the Square 3 : Answers

1) a) $x^2 + 4x - 1 = 0$
 $x^2 + 4x = 1$
 $(x+2)^2 - 4 = 1$

$$(x+2)^2 = 5$$

$$x+2 = \pm\sqrt{5}$$

$$x = -2 \pm \sqrt{5}$$

either $x = 0.24$ or $x = -4.24$

b) $x^2 - 2x - 8 = 0$
 $x^2 - 2x = 8$
 $(x-1)^2 - 1 = 8$
 $(x-1)^2 = 9$
 $x-1 = \pm 3$
 $x = 1 \pm 3$

← This would factorise!
 $(x+2)(x-4) = 0$

either $x = 4$ or $x = -2$

← No decimals needed

c) $x^2 + 3x - 2 = 0$
 $x^2 + 3x = 2$
 $(x + \frac{3}{2})^2 - \frac{9}{4} = 2$
 $(x + \frac{3}{2})^2 = \frac{17}{4}$

$$x + \frac{3}{2} = \pm \frac{\sqrt{17}}{2}$$

$$x = -\frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

either $x = -3.56$ or $x = 0.56$

d) $2x^2 - 6x + 4 = 0$
 $\div 2$ $x^2 - 3x + 2 = 0$
 $x^2 - 3x = -2$
 $(x - \frac{3}{2})^2 - \frac{9}{4} = -2$

$$(x - \frac{3}{2})^2 = \frac{1}{4}$$

$$x - \frac{3}{2} = \pm \frac{1}{2}$$

$$x = \frac{3}{2} \pm \frac{1}{2}$$

← Will factorise $(x-2)(x-1)$

either
 $x = 2$ or $x = 1$

$$e) \quad 2x^2 + 9x - 1 = 0$$

$$2 \left[x^2 + \frac{9}{2}x - \frac{1}{2} \right] = 0$$

$$2 \left[\left(x + \frac{9}{4} \right)^2 - \left(\frac{9}{4} \right)^2 - \frac{1}{2} \right] = 0$$

$$\underline{\times 2} \quad \left(x + \frac{9}{4} \right)^2 - \frac{81}{16} - \frac{1}{2} = 0$$

$$\left(x + \frac{9}{4} \right)^2 = \frac{89}{16}$$

$$x + \frac{9}{4} = \pm \frac{\sqrt{89}}{4}$$

$$x = -\frac{9}{4} \pm \frac{\sqrt{89}}{4}$$

$$\text{either } x = 0.11 \quad \text{or} \quad x = 4.61$$

$$2) \quad ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$\div a$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\left(x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$$

$$\left(x + \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$\left(x + \frac{b}{2a} \right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

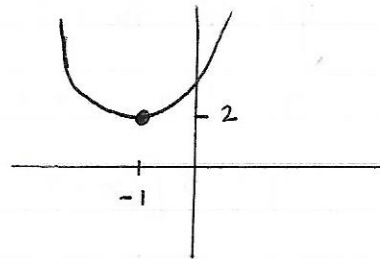
3)

a)

$$y = x^2 + 2x + 3$$

$$y = (x+1)^2 - 1 + 3$$

$$y = (x+1)^2 + 2$$

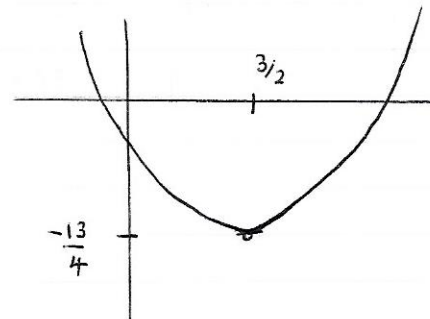


b)

$$y = x^2 - 3x - 1$$

$$y = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 1$$

$$y = \left(x - \frac{3}{2}\right)^2 - \frac{13}{4}$$



c)

$$y = -x^2 + x + 2$$

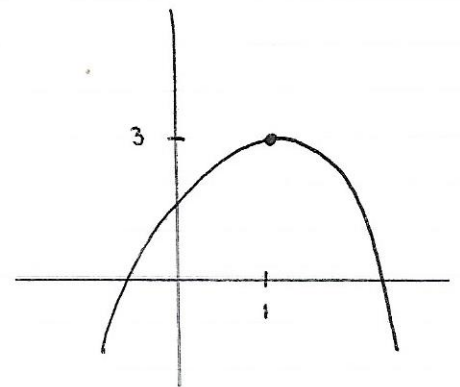
$$y = -(x^2 - x - 2)$$

$$y = -[(x-1)^2 - 1 - 2]$$

$$y = -[(x-1)^2 - 3]$$

$$y = -(x-1)^2 + 3$$

^ ^ ^
1 right 3 up



d)

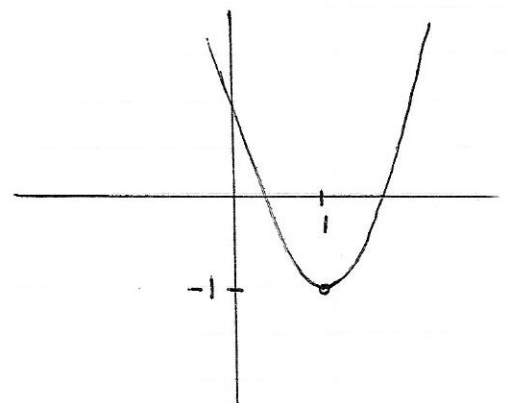
$$y = 2x^2 - 4x + 1$$

$$y = 2\left[x^2 - 2x + \frac{1}{2}\right]$$

$$y = 2\left[(x-1)^2 - 1 + \frac{1}{2}\right]$$

$$y = 2\left[(x-1)^2 - \frac{1}{2}\right]$$

$$y = 2(x-1)^2 - 1$$



e)

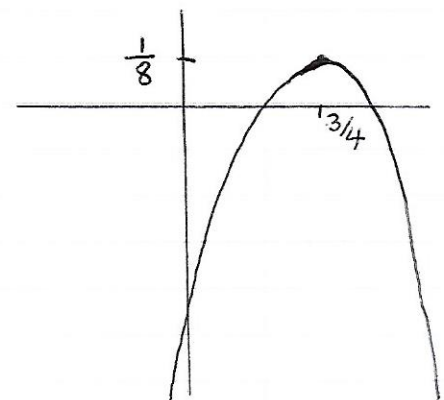
$$y = -2x^2 + 3x - 1$$

$$y = -2\left[x^2 - \frac{3}{2}x + \frac{1}{2}\right]$$

$$y = -2\left[\left(x - \frac{3}{4}\right)^2 - \frac{9}{16} + \frac{1}{2}\right]$$

$$y = -2\left[\left(x - \frac{3}{4}\right)^2 - \frac{1}{16}\right]$$

$$y = -2\left(x - \frac{3}{4}\right)^2 + \frac{1}{8}$$



$$f) \quad y = 3x^2 - 5x + 2$$

$$y = 3\left(x^2 - \frac{5}{3}x + \frac{2}{3}\right)$$

$$y = 3\left[\left(x - \frac{5}{6}\right)^2 - \frac{25}{36} + \frac{2}{3}\right]$$

$$y = 3\left[\left(x - \frac{5}{6}\right)^2 - \frac{1}{36}\right]$$

$$y = 3\left(x - \frac{5}{6}\right)^2 - \frac{1}{12}$$

