Geometric Progressions 1 : Answers

31) a) Proof of $S_n = \frac{a(1-r^n)}{1-r}$ $\quad |r| < 1$

Sum to infinity is $\frac{a}{1-r}$

b)(i) $T_1 + T_2 = 2(T_2 + T_3)$

$$a + ar = 2(ar + ar^2)$$

$\div a$

$$1 + r = 2(r + r^2)$$
$$1 + r = 2r + 2r^2$$
$$0 = 2r^2 + r - 1$$
$$0 = (2r-1)(r+1)$$

$\therefore$ either $2r - 1 = 0$ or $\cancel{r + 1 = 0}$

$r = \frac{1}{2}$ $\quad$ $\cancel{r = -1}$

r is +ve $\therefore$ $r = \frac{1}{2}$

(ii) $S_\infty = 12$

$$12 = \frac{a}{1-r}$$
$$12 = \frac{a}{1-\frac{1}{2}}$$
$$12 = \frac{a}{\frac{1}{2}}$$
$$6 = a$$

Now $S_8 = \frac{6\left(1-\left(\frac{1}{2}\right)^8\right)}{1-\frac{1}{2}}$

$$S_8 = \frac{6\left(1-\frac{1}{256}\right)}{\frac{1}{2}}$$

$$S_8 = 12\left(\frac{255}{256}\right)$$

$$S_8 = 11 \cdot 95 \text{ to 2 d.p.}$$

32) a) Proof $\quad S_n = \dfrac{a(1-r^n)}{1-r} \qquad |r| < 1$

Sum to infinity is $\dfrac{a}{1-r}$

b) (i) $S_\infty = 10$ $\qquad\qquad$ $S_\infty = 15$

$\dfrac{a}{1-r} = 10 \quad — ①$ $\qquad\qquad$ $\dfrac{a}{1-2r} = 15 \quad — ②$

(*) $\quad a = 10(1-r)$

Subst (*) into ②

$$② \Rightarrow \quad \frac{10(1-r)}{(1-2r)} = 15$$

$$2(1-r) = 3(1-2r)$$
$$2-2r = 3-6r$$
$$4r = 1$$
$$r = 1/4$$

also
$$a = 10(1-r)$$
$$a = 10(1-1/4)$$
$$a = 10(3/4)$$
$$a = 15/2$$

(ii) $$S_4 = \frac{\frac{15}{2}\left(1-\left(\frac{1}{4}\right)^4\right)}{1-1/4}$$

$$S_4 = \frac{7.5\left(1-\frac{1}{256}\right)}{3/4}$$

$$S_4 = 9.96 \text{ to } 2 \text{ d.p.}$$

33) $T_5 = 135$ $\qquad T_8 = 5$

$$(1) - \quad ar^4 = 135 \qquad ar^7 = 5 \quad - (2)$$

$\div$ (2) by (1)

(a)

$$\frac{ar^7}{ar^4} = \frac{5}{135}$$

$$r^3 = \frac{1}{27}$$

$$r = \frac{1}{3}$$

(1) $\Rightarrow$

$$a\left(\frac{1}{3}\right)^4 = 135$$

$$a\left(\frac{1}{81}\right) = 135$$

$$a = 135 \times 81$$

$$a = 10935$$

b)

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{10935}{1 - 1/3}$$

$$S_\infty = 16402 \cdot 5 \qquad \text{OR} \qquad \frac{32805}{2}$$

34) $a + ar = 7 \cdot 2$ — ① $S_\infty = 20$

$$\frac{a}{1-r} = 20 \quad — ②$$

$$a = 20(1-r) \quad (*)$$

Sub $(*)$ into ①

$$① \Rightarrow \quad 20(1-r) + 20(1-r)r = 7 \cdot 2$$

$$20 - \cancel{20r} + \cancel{20r} - 20r^2 = 7 \cdot 2$$

$$0 = 20r^2 - 12 \cdot 8$$

$$12 \cdot 8 = 20r^2$$

$$0 \cdot 64 = r^2$$

$$\pm 0 \cdot 8 = r$$

but r is +ve $\therefore$ $r = 0 \cdot 8$

Now $(*)$ $a = 20(1 - 0 \cdot 8)$

$$a = 20(0 \cdot 2)$$

$$a = 4$$