Geometric Progressions 2 : Answers

35) a) Book Work

b) $10 + 9 + 8 \cdot 1 + \ldots$

$a = 10 \qquad r = \frac{9}{10}$

$$S_{18} = \frac{10\left(1 - 0 \cdot 9^{18}\right)}{(1 - 0 \cdot 9)}$$

$$S_{18} = \frac{10\,(0 \cdot 8499)}{0 \cdot 1}$$

$$S_{18} = 84 \cdot 99 \approx 85$$

c) $ar = -4 \qquad S_\infty = 9$

(1) — $a = \frac{-4}{r} \qquad \frac{a}{1-r} = 9$

$$a = 9(1-r) \quad — (2)$$

Sub (1) into (2)

$$\frac{-4}{r} = 9(1-r)$$

$$-4 = 9r(1-r)$$

$$-4 = 9r - 9r^2$$

(i) $9r^2 - 9r - 4 = 0$

$(3r - 4)(3r + 1) = 0$

either $3r - 4 = 0$ or $3r + 1 = 0$

$r = 4/3$ $\qquad r = -\frac{1}{3}$

ii) $\therefore \quad r = -\frac{1}{3}$ because $|r| < 1$ from the fact that S_∞ is finite.

36) a) $T_9 = 36 \quad T_{10} = 108$

① — $ar^8 = 36 \quad ar^9 = 108$ — ②

$$\div \quad \frac{ar^9}{ar^8} = \frac{108}{36}$$

$$r = 3$$

$$\therefore \ ① \Rightarrow \quad a(3^8) = 36$$

$$a = \frac{36}{3^8}$$

$$a = \frac{36}{6561}$$

$$a = \frac{4}{729}$$

$$\therefore \quad T_7 = ar^6$$

$$T_7 = \frac{4}{729} \times 3^6$$

$$T_7 = 4$$

b) $T_2 = 9 \qquad S_\infty = 48$

$$ar = 9 \qquad \frac{a}{1-r} = 48 \quad — \ ②$$

① — $a = \frac{9}{r}$

Sub ① into ②

$$② \Rightarrow \quad \frac{\frac{9}{r}}{(1-r)} = 48$$

(i)

$$\frac{9}{r} = 48(1-r)$$

$$9 = 48r(1-r)$$

$$9 = 48r - 48r^2$$

$$48r^2 - 48r + 9 = 0$$

$$16r^2 - 16r + 3 = 0 \quad \ldots \text{ QED}$$

(ii) $16r^2 - 16r + 3 = 0$

$(4r - 3)(4r - 1) = 0$

either $4r - 3 = 0$

$r = \frac{3}{4}$

or $4r - 1 = 0$

$r = \frac{1}{4}$

from ① $a = \frac{9}{3/4}$ $\quad a = \frac{9}{1/4}$

$a = \frac{9 \times 4}{3}$ $\quad a = 9 \times 4$

$a = 12$ $\quad a = 36$

37) a) Book work

b) $S_4 = 73 \cdot 8$ $\quad S_\infty = 125$

$\frac{a(1 - r^4)}{(1 - r)} = 73 \cdot 8$ $\quad \frac{a}{(1 - r)} = 125$ — ②

① — $a = \frac{73 \cdot 8(1 - r)}{(1 - r^4)}$

Sub ① into ②

$$\frac{\frac{73 \cdot 8(\cancel{1 - r})}{(1 - r^4)}}{\cancel{(1 - r)}} = 125$$

$73 \cdot 8 = 125(1 - r^4)$

$73 \cdot 8 = 125 - 125r^4$

$125r^4 = 51 \cdot 2$

$r^4 = \frac{51 \cdot 2}{125}$

$r = 0 \cdot 8$ or $\frac{4}{5}$

$\therefore a = \frac{73 \cdot 8(1 - 0 \cdot 8)}{(1 - 0 \cdot 8^4)} = 25$

38) a) $40 - 24 + 14.4 - \ldots$

$a = 40 \qquad r = \frac{-24}{40} = -\frac{3}{5} = -0.6$

$S_\infty = \frac{a}{1-r} = \frac{40}{1-(-0.6)} = \frac{40}{1.6} = 25$

b) $T_4 = 8$

$T_3 + T_4 + T_5 = 28$

$ar^3 = 8 \qquad ar^2 + 8 + ar^4 = 28$

$a = \frac{8}{r^3}$ — (1) $\qquad ar^2 + ar^4 = 20$ — (2)

Sub (1) into (2)

(2) $\Rightarrow \quad \frac{8}{r^3} r^2 + \frac{8}{r^3} r^4 = 20$

$\frac{8}{r} + 8r = 20$

$8 + 8r^2 = 20r$

(i) $2r^2 - 5r + 2 = 0 \quad \ldots$ QED

$(2r - 1)(r - 2) = 0$

either $r = \frac{1}{2}$ or $r = 2$

(ii) $\therefore \quad r = \frac{1}{2}$ as $|r| < 1$

$\therefore$ (1) $\Rightarrow \quad a = \frac{8}{(\frac{1}{2})^3}$

$a = 8 \times 8$

$a = 64$