Geometric Progressions 3: Answers

39) a) $T_2 = 6$ $\quad T_5 = 384$

(i) $ar = 6$ — ① $\quad ar^4 = 384$ — ②

÷ ② by ① $\quad \dfrac{ar^4}{ar} = \dfrac{384}{6}$

$$r^3 = 64$$
$$r = 4$$

$\therefore$ ① $\Rightarrow$ $a = \dfrac{6}{r}$

$$a = \frac{6}{4} = \frac{3}{2} = 1.5$$

(ii) $$S_8 = \frac{a(r^8 - 1)}{(r-1)}$$

$$S_8 = \frac{1.5(4^8 - 1)}{(4-1)}$$

$$S_8 = \frac{1.5(65535)}{3}$$

$$S_8 = 32767.5$$

b) $a = 5 \quad r = 1.1$

(i) $T_n = 170$

$$ar^{n-1} = 170$$
$$(5)\,1.1^{n-1} = 170$$
$$1.1^{n-1} = 34$$

$$\ln 1.1^{n-1} = \ln 34$$
$$(n-1)\ln 1.1 = \ln 34$$
$$n - 1 = \frac{\ln 34}{\ln 1.1}$$

$$n = \frac{\ln 34}{\ln 1.1} + 1$$
$$n = 38$$

(ii) Dafydd is wrong. Because $r > 1$ the S_∞ value will not be finite $S_\infty \to \infty$

40) a) Book Work

b) (i) $S_\infty = 4a$

$$\frac{a}{1-r} = 4a$$

$$\frac{1}{1-r} = 4$$

$$1 = 4(1-r)$$

$$1 = 4 - 4r$$

$$4r = 3$$

$$r = \frac{3}{4}$$

(ii) $a + ar = 35$

$$a + \frac{3}{4}a = 35$$

$$\frac{7}{4}a = 35$$

$$a = \frac{140}{7} = 20$$

$$\therefore \quad S_9 = \frac{20(1 - 0.75^9)}{(1 - 0.75)} = 74$$

41) a) Book Work

b) $a + ar = 25.2$ — (1) $\quad S_\infty = 30$

$$\frac{a}{1-r} = 30$$

$$a = 30(1-r) \quad \text{— (2)}$$

sub (2) into (1)

$$(1) \Rightarrow \quad 30(1-r) + 30r(1-r) = 25.2$$

$$30 - \cancel{30r} + \cancel{30r} - 30r^2 = 25.2$$

$$0 = 30r^2 - 4.8$$

$$\sqrt{\frac{4.8}{30}} = r$$

$$\pm \frac{2}{5} = r$$

$$\therefore \quad r = 2/5 = 0.4 \qquad \left(r \text{ is } +\text{ve}\right)$$

$$\therefore \quad (2) \Rightarrow \quad a = 30(1 - 0.4)$$

$$a = 30(0.6)$$

$$a = 18$$

42) $a + ar = 72$ $\qquad a + ar^2 = 120$ — (2)

$a(1+r) = 72$

$a = \frac{72}{(1+r)}$ — (1)

sub (1) into (2)

(2) $\Rightarrow \frac{72}{1+r} + \frac{72}{1+r} r^2 = 120$

$\times(1+r)$ $\quad 72 + 72r^2 = 120(1+r)$

$72 + 72r^2 = 120 + 120r$

$72r^2 - 120r - 48 = 0$

$\div 24$ $\quad 3r^2 - 5r - 2 = 0$

QED.

b) $3r^2 - 5r - 2 = 0$

$(3r+1)(r-2) = 0$

either $r = -\frac{1}{3}$ or $r = 2$

Now $|r| < 1$ $\quad \therefore$ $r = -\frac{1}{3}$

(1) $\Rightarrow a = \frac{72}{1 - \frac{1}{3}} = \frac{72}{\frac{2}{3}}$

$a = 72 \times \frac{3}{2}$

$a = 36 \times 3$

$a = 108$

$\therefore$ $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{108}{1+\frac{1}{3}} = \frac{108}{\frac{4}{3}} = 108 \times \frac{3}{4}$

$S_\infty = 81$