Completing the Square 4 : Answers

1) a) $x^2 + 4x + 1$

$= (x+2)^2 - 4 + 1$

$= (x+2)^2 - 3$ Min $= -3$ when $x = -2$

b) $x^2 - 6x - 2$

$= (x-3)^2 - 9 - 2$

$= (x-3)^2 - 11$ Min $= -11$ when $x = 3$

c) $x^2 + x - 3$

$= (x + \frac{1}{2})^2 - \frac{1}{4} - 3$

$= (x + \frac{1}{2})^2 - \frac{13}{4}$ Min $= -\frac{13}{4}$ when $x = -\frac{1}{2}$

d) $2x^2 - 4x + 1$

$= 2[x^2 - 2x + \frac{1}{2}]$

$= 2[(x-1)^2 - 1 + \frac{1}{2}]$

$= 2[(x-1)^2 - \frac{1}{2}]$ Min $= -1$ when $x = 1$

$= 2(x-1)^2 - 1$

e) $2x^2 + 3x - 1$

$= 2[x^2 + \frac{3}{2}x - \frac{1}{2}]$

$= 2[(x + \frac{3}{4})^2 - \frac{9}{16} - \frac{1}{2}]$

$= 2[(x + \frac{3}{4})^2 - \frac{17}{16}]$ Min $= -\frac{17}{8}$ when $x = -\frac{3}{4}$

$= 2(x + \frac{3}{4})^2 - \frac{17}{8}$

f) $3x^2 - 2x + 5$

$= 3[x^2 - \frac{2}{3}x + \frac{5}{3}]$

$= 3[(x - \frac{1}{3})^2 - \frac{1}{9} + \frac{5}{3}]$

$= 3[(x - \frac{1}{3})^2 + \frac{2}{3}]$

$= 3(x - \frac{1}{3})^2 + 2$ Min $= 2$ when $x = \frac{1}{3}$

2) a) $-x^2 - 2x + 6$

$= -[x^2 + 2x - 6]$

$= -[(x+1)^2 - 1 - 6]$

$= -[(x+1)^2 - 7]$

$= -(x+1)^2 + 7$

Max = 7 when $x = -1$

b) $-2x^2 + 4x + 1$

$= -2\left[x^2 - 2x - \frac{1}{2}\right]$

$= -2\left[(x-1)^2 - 1 - \frac{1}{2}\right]$

$= -2\left[(x-1)^2 - \frac{3}{2}\right]$

$= -2(x-1)^2 + 3$

Max = 3 when $x = 1$

c) $-2x^2 - 3x - 1$

$= -2\left[x^2 + \frac{3}{2}x + \frac{1}{2}\right]$

$= -2\left[\left(x + \frac{3}{4}\right)^2 - \frac{9}{16} + \frac{1}{2}\right]$

$= -2\left[\left(x + \frac{3}{4}\right)^2 - \frac{1}{16}\right]$

$= -2\left(x + \frac{3}{4}\right)^2 + \frac{1}{8}$

Max = $\frac{1}{8}$ when $x = -\frac{3}{4}$

d) $-3x^2 + x + 2$

$= -3\left(x^2 - \frac{1}{3}x - \frac{2}{3}\right)$

$= -3\left[\left(x - \frac{1}{6}\right)^2 - \frac{1}{36} - \frac{2}{3}\right]$

$= -3\left[\left(x - \frac{1}{6}\right)^2 - \frac{25}{36}\right]$

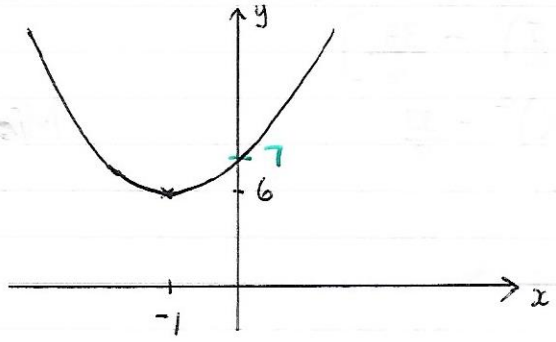
$= -3\left(x - \frac{1}{6}\right)^2 + \frac{25}{12}$

Max = $\frac{25}{12}$ when $x = \frac{1}{6}$

3) a) $y = x^2 + 2x + 7$ ← cross y axis

$y = (x+1)^2 - 1 + 7$

$y = (x+1)^2 + 6$

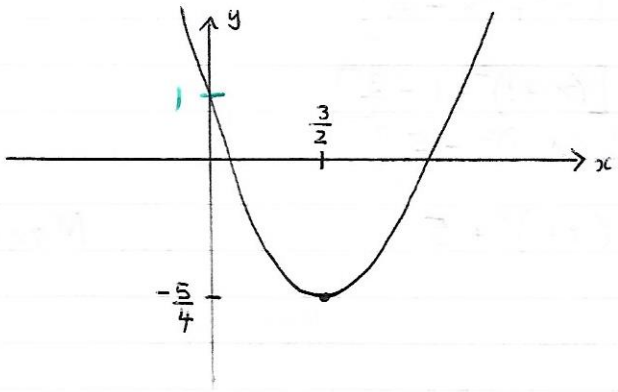


Min = 6 when $x = -1$

b) $y = x^2 - 3x + 1$ ← cross y axis

$y = (x - \frac{3}{2})^2 - \frac{9}{4} + 1$

$y = (x - \frac{3}{2})^2 - \frac{5}{4}$



Min = $-\frac{5}{4}$ when $x = \frac{3}{2}$

c) $y = -x^2 + 2x - 3$ ← cross y axis

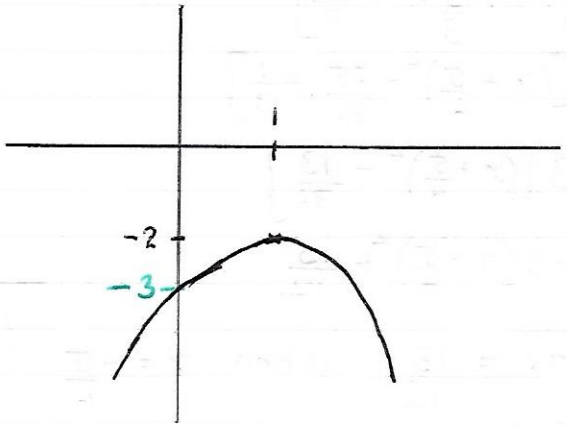
$y = -[x^2 - 2x + 3]$

$y = -[(x-1)^2 - 1 + 3]$

$y = -[(x-1)^2 + 2]$

$y = -(x-1)^2 - 2$

Max = -2 when $x = 1$



d) $y = 2x^2 + 5x - 1$ ← cross y axis

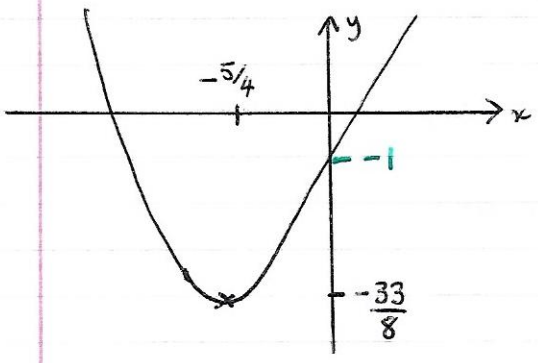
$y = 2\left[x^2 + \frac{5}{2}x - \frac{1}{2}\right]$

$y = 2\left[\left(x + \frac{5}{4}\right)^2 - \frac{25}{16} - \frac{1}{2}\right]$

$y = 2\left[\left(x + \frac{5}{4}\right)^2 - \frac{33}{16}\right]$

$y = 2\left(x + 5/4\right)^2 - \frac{33}{8}$

Min $= -\frac{33}{8}$ when $x = -\frac{5}{4}$



e) $y = -2x^2 - 4x + 3$ ← cross y axis

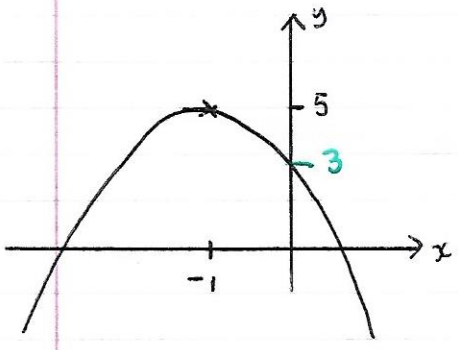
$y = -2\left[x^2 + 2x - \frac{3}{2}\right]$

$y = -2\left[(x+1)^2 - 1 - \frac{3}{2}\right]$

$y = -2\left[(x+1)^2 - \frac{5}{2}\right]$

$y = -2(x+1)^2 + 5$

Max $= 5$ when $x = -1$



f) $y = -3x^2 - 5x - 1$ ← cross y axis

$y = -3\left[x^2 + \frac{5}{3}x + \frac{1}{3}\right]$

$y = -3\left[\left(x + \frac{5}{6}\right)^2 - \frac{25}{36} + \frac{1}{3}\right]$

$y = -3\left[\left(x + \frac{5}{6}\right)^2 - \frac{13}{36}\right]$

$y = -3\left(x + \frac{5}{6}\right)^2 + \frac{13}{12}$

Max $= \frac{13}{12}$ when $x = -\frac{5}{6}$