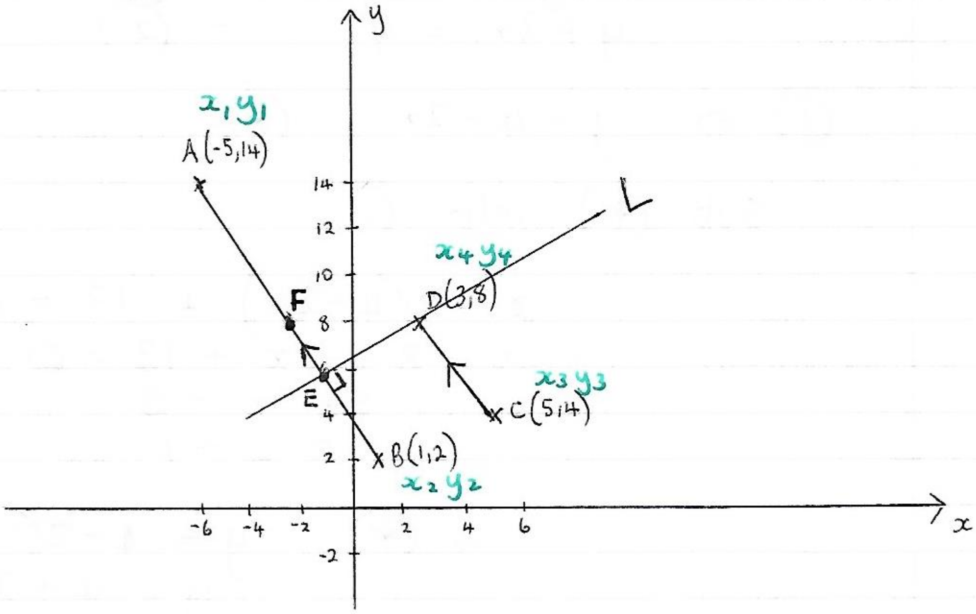
Coordinate Geometry 5 : Answers

12)



a) (i) AB

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{2-14}{1+5} = \frac{-12}{6} = -2$

CD

$m = \frac{y_4 - y_3}{x_4 - x_3}$

$m = \frac{8-4}{3-5} = \frac{4}{-2} = -2$

$\therefore$ AB and CD are parallel, they have the same gradients. .. QED

ii) AB

$m = -2$ Use $A(-5,14)$

$$y - y_1 = m(x - x_1)$$
$$y - 14 = -2(x+5)$$
$$y - 14 = -2x - 10$$
$$y + 2x = 4$$

iii) L

Grad $= +\frac{1}{2}$ because $\perp$ to AB

$(+\frac{1}{2}) \times (-2) = -1$

Use $D(3,8)$

Eqn of L $\quad y - y_4 = m(x - x_4)$

$$y - 8 = \frac{1}{2}(x-3)$$
$$2y - 16 = x - 3$$
$$0 = x - 2y + 13$$ ... QED

b) (i) Solve $x - 2y + 13 = 0$ — ①

$y + 2x = 4$ — ②

② $\Rightarrow$ $y = 4 - 2x$ $(*)$

Sub $(*)$ into ①

$$x - 2(4-2x) + 13 = 0$$
$$x - 8 + 4x + 13 = 0$$
$$5x = -5$$
$$x = -1$$

$\therefore (*)$ $y = 4 - 2(-1)$

$$y = 4 + 2$$
$$y = 6$$

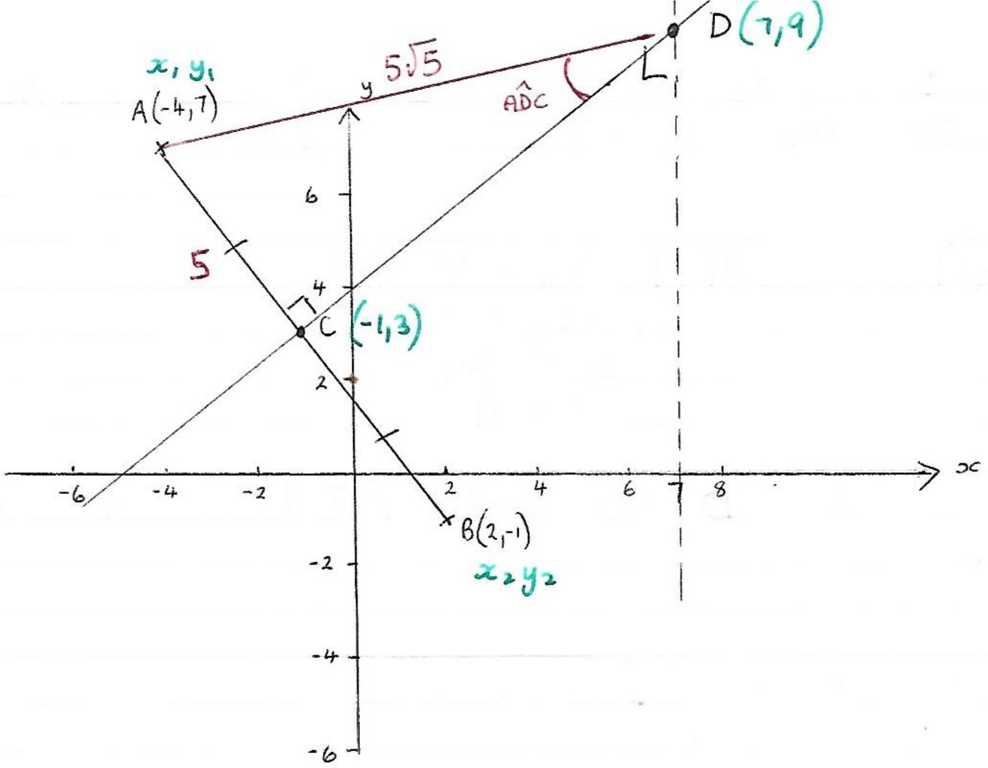
$\therefore$ $E(\underset{x_5}{-1}, \underset{y_5}{6})$

(ii) F is midpoint AB

$$F\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= F\left(\frac{-5+1}{2}, \frac{14+2}{2}\right)$$
$$= F(\underset{x_6}{-2}, \underset{y_6}{8})$$

$$EF = \sqrt{(x_6-x_5)^2 + (y_6-y_5)^2}$$
$$= \sqrt{(-2+1)^2 + (8-6)^2}$$
$$= \sqrt{(-1)^2 + (2)^2}$$
$$= \sqrt{1+4}$$
$$= \sqrt{5} \text{ units.}$$

13)



a) AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1-7}{2+4} = \frac{-8}{6} = -\frac{4}{3}$$

b) C

$$C\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= C\left(\frac{-4+2}{2}, \frac{7-1}{2}\right)$$

$$= C(-1, 3)$$

$x_3 \; y_3$

c) L

$m = +\frac{3}{4}$ because it is $\perp$ to AB

$$\left(+\frac{3}{4}\right) \times \left(-\frac{4}{3}\right) = -1$$

Use $C(-1,3)$

Eqn L

$$y - y_3 = m(x - x_3)$$

$$y - 3 = \frac{3}{4}(x+1)$$

$$4y - 12 = 3(x+1)$$

$$4y - 12 = 3x + 3$$

$$0 = 3x - 4y + 15$$ ... QED.

d) L $\quad 3x - 4y + 15 = 0$

Also line $\quad x = 7$ $\quad \Big\}$ Solve simultaneously

(i)

$$3(7) - 4y + 15 = 0$$
$$21 - 4y + 15 = 0$$
$$36 = 4y$$
$$9 = y$$

$\therefore$ D has coords $(\underset{x_4}{7}, \underset{y_4}{9})$ $\quad \underline{\underline{ie}}$ $\quad k = 9$

QED.

(ii)

$$CA = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$= \sqrt{(-1+4)^2 + (3-7)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25} = 5 \text{ units}$$

$$DA = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$$
$$= \sqrt{(7+4)^2 + (9-7)^2}$$
$$= \sqrt{11^2 + 2^2}$$
$$= \sqrt{121 + 4} = \sqrt{125} = 5\sqrt{5} \text{ units}$$

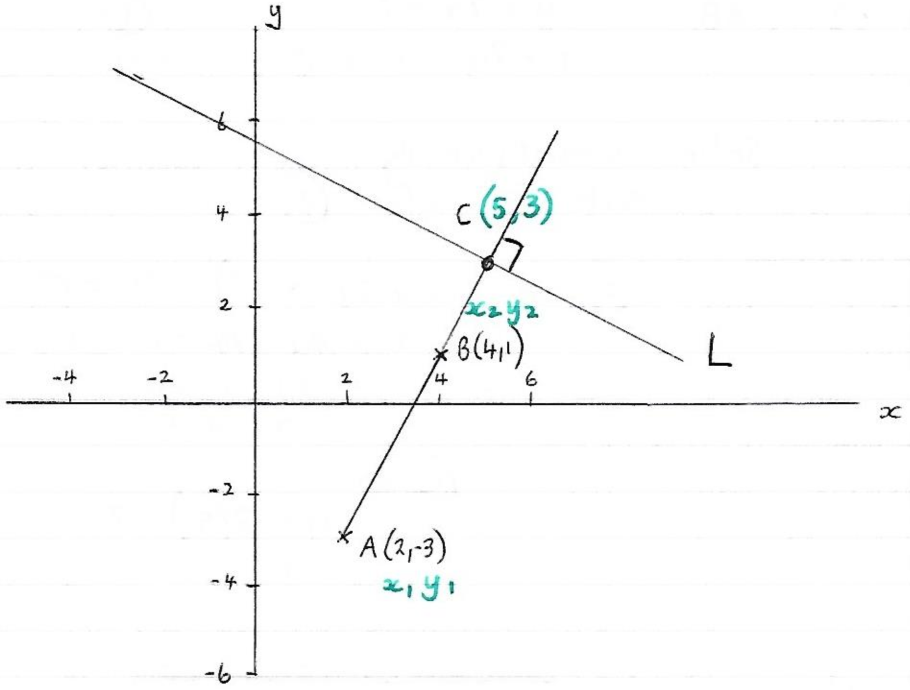
(iii)

from SOHCAHTOA on triangle ACD

$$\sin A\hat{D}C = \frac{5}{5\sqrt{5}} = \frac{1}{\sqrt{5}}$$

where $a = 5$

14)



a) AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1+3}{4-2} = \frac{4}{2} = +2$$

Eqn. AB   Use $A(2,-3)$

$$y - y_1 = m(x - x_1)$$
$$y + 3 = 2(x - 2)$$
$$y + 3 = 2x - 4$$
$$y = 2x - 7$$

b) L $x + 2y - 11 = 0$

$$2y = -x + 11$$
$$y = -\frac{1}{2}x + \frac{11}{2}$$

$$\therefore \quad m = -\frac{1}{2}$$

$\therefore$ L is $\perp$ to AB because

$$\left(-\frac{1}{2}\right) \times (+2) = -1$$

c) AB $y = 2x - 7$ — ①

L $x + 2y - 11 = 0$ — ②

Solve simultaneously

Sub ① into ②

② $\Rightarrow$ $x + 2(2x - 7) - 11 = 0$

$x + 4x - 14 - 11 = 0$

$5x = 25$

$x = 5$

$\therefore$ ① $\Rightarrow$ $y = 2(5) - 7$

$y = 3$

$x_3 \; y_3$

$\therefore$ $C(5, 3)$

... QED.

d) $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-2)^2 + (1+3)^2}$

$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$AC = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$= \sqrt{(5-2)^2 + (3+3)^2}$

$= \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$

Now $AB = kAC$ find k

$2\sqrt{5} = k \times 3\sqrt{5}$

$\div\sqrt{5}$

$2 = 3k$

$\frac{2}{3} = k$