Inflexion Points 1 : Answers

$$y = ax^4 + bx^3 + 18x^2$$

a) $\frac{dy}{dx} = 4ax^3 + 3bx^2 + 36x$

$\frac{d^2y}{dx^2} = 12ax^2 + 6bx + 36 = 0$ at inflexion points

Now $(1, 11)$ is given as inflexion point

$\therefore$ Let $x = 1$

$$\frac{d^2y}{dx^2} = 12a(1) + 6b(1) + 36 = 0$$

$$2a + b + 6 = 0$$

QED

b) Now $2a + b + 6 = 0$ — ①

Find a second eqn. with a and b.

We know $(1, 11)$ lies on the curve.

$\therefore$ Sub $x = 1$, $y = 11$ into eqn. of curve!

$$11 = a(1)^4 + b(1)^3 + 18(1)^2$$

$$11 = a + b + 18 \quad \text{— ②}$$

Solve ① + ②

① $\Rightarrow$ $b = -6 - 2a$ (*)

$\therefore$ ② $\Rightarrow$ $11 = a - 6 - 2a + 18$

$$a = 1$$

from (*)

$$b = -6 - 2(1)$$
$$b = -8$$

Now need to find other inflection point.

$$\frac{d^2y}{dx^2} = 12ax^2 + 6bx + 36 = 0$$ at inflexion points

$a = 1 \quad b = -8$

$$12(1)x^2 + 6(-8)x + 36 = 0$$
$$12x^2 - 48x + 36 = 0$$
$$\div 12 \quad x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$

either $x = 3$ or $x = 1$

$x = 1 \rightarrow (1, 11)$ GIVEN

$$y = (1)x^4 + (-8)x^3 + 18x^2$$
$$y = (1)(3^4) - 8(3)^3 + 18(3^2)$$
$$y = 81 - 216 + 162$$
$$y = 27$$

$\therefore$ Inflection point $(3, 27)$

c) Now SP's $\quad \frac{dy}{dx} = 4(1)x^3 + 3(-8)x^2 + 36x = 0$ at SPs

$$4x^3 - 24x^2 + 36x = 0$$
$$x^3 - 6x^2 + 9x = 0$$
$$x(x^2 - 6x + 9) = 0$$
$$x(x-3)(x-3) = 0$$

$x = 0$ or $x = 3$

$y = 0 \quad (0,0)$

$(3, 27)$ inflexion point also SP

Now nature of $(0,0)$

$$\frac{d^2y}{dx^2} = 12x^2 - 48x + 36$$

$x = 0$ $\quad \frac{d^2y}{dx^2} = +36 \quad \therefore$ LOCAL MIN $(0,0)$

Sketch