ANSWERS

i)

$$y = -x^3 + 6x^2 - 9x$$

$$\frac{dy}{dx} = -3x^2 + 12x - 9 = 0 \quad \text{at SP's.}$$

$$-x^2 + 4x - 3 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

either $x = 3$ or $x = 1$

$x = 3 \downarrow$

$$y = -27 + 54 - 27$$
$$y = 0$$

$(3,0)$

$x = 1 \downarrow$

$$y = -1 + 6 - 9$$
$$y = -4$$

$(1,-4)$

$$\frac{d^2y}{dx^2} = -6x + 12 = 0 \quad \text{at point of inflexion}$$

$$12 = 6x$$

$$2 = x$$

$\downarrow$

$$y = -8 + 24 - 18$$
$$y = -2$$

$(2,-2)$

NATURE

ALSO at $x = 3$

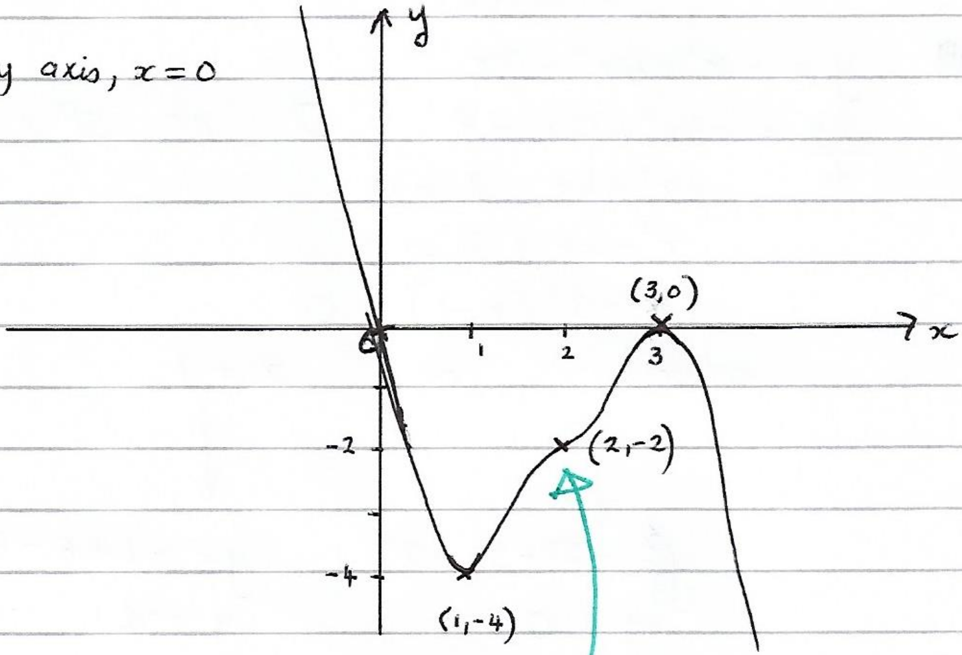
$$\frac{d^2y}{dx^2} = -6(3) + 12 = -ve \qquad (3,0) \text{ Local max}$$

$x = 1$

$$\frac{d^2y}{dx^2} = -6(1) + 12 = +ve \qquad (1,-4) \text{ Local Min}$$

Curve crosses y axis, $x = 0$

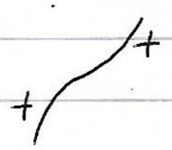
$y = 0$



The shape of the inflexion point is obvious

* It is not a stationary point and must go from the position of the max and min

* You do not need to use $\frac{dy}{dx}$ values to show



in this question

2) $y = x(x-1)(x-2)$

$y = x(x^2 - 3x + 2)$

$y = x^3 - 3x^2 + 2x$

$\frac{dy}{dx} = 3x^2 - 6x + 2$

$\frac{d^2y}{dx^2} = 6x - 6 = 0$ at inflexion points

$x - 1 = 0$

$x = 1$

$\downarrow$

$y = 1(0)(-1)$

$y = 0$

$\therefore$ $(1,0)$ is an inflexion point.

<u>Nature</u>

First $\frac{dy}{dx} = 3x^2 - 6x + 2 = 0$ at SP's

$$x = \frac{6 \pm \sqrt{36 - 4(3)(2)}}{6}$$

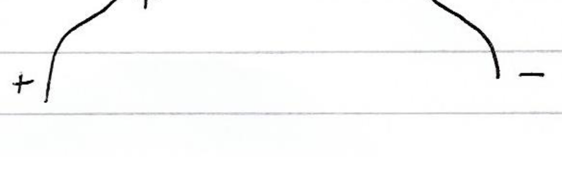
$$x = \frac{6 \pm \sqrt{12}}{6}$$

$$x = \frac{6 \pm 2\sqrt{3}}{6} = \frac{3 \pm \sqrt{3}}{3}$$

There are 2 SP's but <u>neither</u> have an x value of 1.

$\therefore$ $(1,0)$ is an inflexion point but not a S.P.

Now to decide

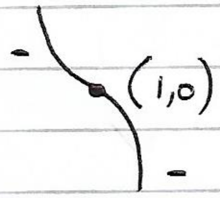


$\underline{x = 0.9}$ $\underline{x = 1.1}$

$$\frac{dy}{dx} = 3(0.9)^2 - 6(0.9) + 2$$
$$= 2.43 - 5.4 + 2$$
$$= -ve$$

$$\frac{dy}{dx} = 3(1.1)^2 - 6(1.1) + 2$$
$$= 3.63 - 6.6 + 2$$
$$= -ve$$

$\therefore$ $(1,0)$ is an inflexion point looking like



3) $y = x^3 + ax^2 + bx + c$

a) At $(0,0)$

$$0 = 0 + 0 + 0 + c$$

$$0 = c \quad — \quad (1)$$

At $(1, 2/3)$

$$2/3 = 1^3 + a(1)^2 + b(1) + 0$$

$$\frac{2}{3} = 1 + a + b$$

$$-1/3 = a + b$$

$$-1 = 3a + 3b \quad — \quad (2)$$

Now

$$\frac{dy}{dx} = 3x^2 + 2ax + b$$

$$\frac{d^2y}{dx^2} = 6x + 2a \quad = 0 \text{ at inflexion points}$$

$(1, 2/3)$ is inflexion point

$\underline{x = 1}$

$$6(1) + 2a = 0$$

$$2a = -6$$

$$a = -3 \quad — \quad (3)$$

From (2)

$$-1 = 3(-3) + 3b$$

$$-1 + 9 = 3b$$

$$8/3 = b \quad — \quad (4)$$

b) Curve is $y = x^3 - 3x^2 + \frac{8}{3}x$

$$\frac{dy}{dx} = 3x^2 - 6x + \frac{8}{3}$$

Now at $(0,0)$ we know

$$\frac{dy}{dx} = 3(0)^2 - 6(0) + 8/3$$

$$\frac{dy}{dx} = \frac{8}{3}$$

$\therefore$ Eqn tangent at $(0,0)$

$$y - 0 = \frac{8}{3}(x - 0)$$

$$y = \frac{8}{3}x$$

$$3y = 8x$$

4) Let $y = f(x)$ be written

$$y = ax^3 + bx^2 + cx + d$$

$$\frac{dy}{dx} = 3ax^2 + 2bx + c$$

$$\frac{d^2y}{dx^2} = 6ax + 2b = 0 \text{ at inflexion}$$

$$3ax + b = 0$$

At $(2, -22)$

$$3a(2) + b = 0$$

$$6a + b = 0 \quad — \ (1)$$

Also $(2, -22)$ lies on the curve

$$\therefore \ -22 = a(8) + b(4) + c(2) + d$$

$$-22 = 8a + 4b + 2c + d \quad — \ (2)$$

Now Eqn tangt is $y + 3x = 0$

$$y = -3x$$

$$\therefore \ \text{grad} = -3$$

$$\therefore \ \frac{dy}{dx} = 3ax^2 + 2bx + c = -3$$

when $x = 0$ at origin

$$0 + 0 + c = -3$$

$$c = -3 \quad — \ (3)$$

Also $(0,0)$ lies on curve

$$0 = 0 + 0 + 0 + d$$

$$0 = d \quad — \ (4)$$

Now (2) $\Rightarrow$ $-22 = 8a + 4b + 2(-3) + 0$

$$-22 = 8a + 4b - 6$$

$$-16 = 8a + 4b$$

$$-4 = 2a + b \quad \text{— } (5)$$

Solve (1) and (5) simultaneously

(1) $\Rightarrow$ $b = -6a$

(2) $\Rightarrow$ $-4 = 2a - 6a$

$$-4 = -4a$$

$$\underline{1 = a}.$$

$\therefore$ (1) $\Rightarrow$ $b = -6(1)$

$$\underline{b = -6}.$$

$\therefore$ Curve is $y = x^3 - 6x^2 - 3x$

~~Now~~ $\cancel{\frac{dy}{dx} = 3x^2 - 12x - 3 = 0}$ ~~at sps.~~

$\cancel{x^2 - 4x - 1 = 0}$