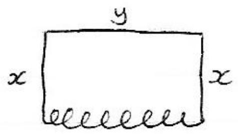
Optimisation : Answers

1)



First $\quad x + y + x = 100$

$$y = 100 - 2x \quad — \quad (1)$$

Now we want max area so find area

$$A = lw \quad \text{for a rectangle}$$

$$A = yx$$

sub (1) in $\quad A = (100-2x)x$

$$A = 100x - 2x^2$$

← We now have area formula in terms of just ONE variable x

$\therefore \quad \dfrac{dA}{dx} = 100 - 4x = 0$ at SP's

$$100 = 4x$$

$$25 = x$$

$\therefore \quad y = 100 - 2(25)$

$$y = 50$$

Now

$$\frac{d^2A}{dx^2} = -4$$

which is <u>always</u> −ve for any x value

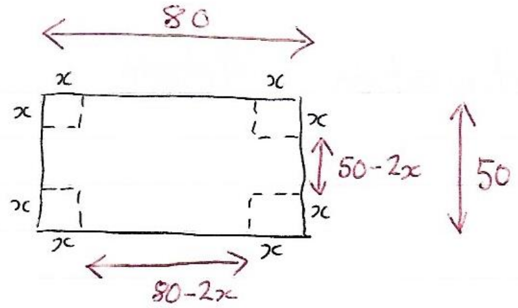
$\therefore$ This is a <u>max</u>

$\therefore$ Max area possible $= 50 \times 25$

$$= 1250 \text{ M}^2$$

when $l = 50\text{m} \quad w = 25\text{m}$.

2)



$V = lwh$ is volume of cuboid

$V = (80-2x)(50-2x)x$

$V = (4000 - 160x - 100x + 4x^2)x$

$V = 4000x - 260x^2 + 4x^3$

We have volume formula in terms of ONE variable x

$\frac{dV}{dx} = 4000 - 520x + 12x^2 = 0$ at SP's

$\div 4$

$3x^2 - 130x + 1000 = 0$

$(3x - 100)(x - 10) = 0$

either $x = \frac{100}{3}$ or $x = 10$

$x = \cancel{33\tfrac{1}{3}}$

IMPOSSIBLE from diagram

Now $\frac{d^2V}{dx^2} = -520 + 24x$

At $x = 10$

$\frac{d^2V}{dx^2} = -520 + 240$

$= -280$ $\therefore$ Local MAX

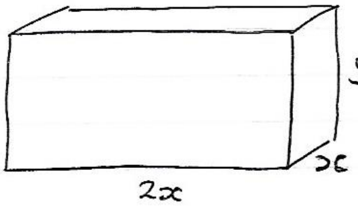
$\therefore$ Max volume possible when $x = 10$ cm

Max $V = (80-20)(50-20)10$

Max $V = (60)(30)10$

Max $V = 18\,000$ cm^3

3)



introduce this yourself

First
$$V = lwh$$
$$9000 = 2x(x)(y)$$
$$9000 = 2x^2y$$
$$\frac{9000}{2x^2} = y$$
$$\frac{4500}{x^2} = y \quad \text{—— } (1)$$

we know this

a) Now we want expression for S

$$S = \underbrace{2(2x)(x)}_{\text{Top + Bottom faces}} + \underbrace{2(2x)y}_{\text{front + back faces}} + \underbrace{2xy}_{\text{left + right faces}}$$

Sub (1) in here
$$S = 4x^2 + 4x\left(\frac{4500}{x^2}\right) + 2x\left(\frac{4500}{x^2}\right)$$

$$S = 4x^2 + \frac{18000}{x} + \frac{9000}{x}$$

$$S = 4x^2 + \frac{27000}{x}$$ ← we now have S in terms of ONE variable x

$27000x^{-1}$

b)
$$\frac{dS}{dx} = 8x - \frac{27000}{x^2}$$

$$\therefore \; 8x - \frac{27000}{x^2} = 0 \quad \text{at SP's}$$

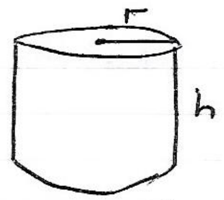
$\times x^2$
$$8x^3 - 27000 = 0$$
$$x^3 = \frac{27000}{8}$$
$$x = \frac{30}{2} = 15 \text{ cm}$$

Now
$$\frac{d^2S}{dx^2} = 8 + \frac{54000}{x^3}$$

$x = 15$
$$\frac{d^2S}{dx^2} = +\text{ve value} \; \therefore \text{Min Value.}$$

$$\therefore \text{ Min S possible} = 4(15)^2 + \frac{27000}{15}$$
$$= 2700 \text{ cm}^2$$ when $x = 15$ cm.

4)



$$V = \pi r^2 h$$
$$440 = \pi r^2 h$$
$$\frac{440}{\pi r^2} = h \quad — \ 1$$

Now we want min metal used or min area

$$A = \underbrace{\pi r^2}_{\text{TOP}} + \underbrace{\pi r^2}_{\text{BOTTOM}} + \underbrace{2\pi r h}_{\text{curved surface}}$$

sub 1 in here

$$A = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{440}{\cancel{\pi} r^{\cancel{2}}}\right)$$

$$A = 2\pi r^2 + \frac{880}{r}$$ ← We have A in terms of ONE variable r

$$\frac{dA}{dr} = 4\pi r - \frac{880}{r^2} = 0 \quad \text{at SP's}$$

$$4\pi r^3 - 880 = 0$$
$$r^3 = \frac{880}{4\pi}$$
$$r = 4 \cdot 12 \text{ cm}$$

$$\therefore \frac{d^2A}{dr^2} = 4\pi + \frac{1760}{r^3}$$

$r = 4 \cdot 12$ $\quad \frac{d^2A}{dr^2} = +ve \quad \therefore$ Min value

$\therefore$ We are not asked to find this min area.

We are asked for the dimensions to give min area

$$r = 4 \cdot 12 \text{ cm}$$

$$h = \frac{440}{\pi (4 \cdot 12)^2} = 8 \cdot 25 \text{cm}.$$

(5) * This is a TRICKY Question

first $C = V^2 + \frac{4000}{V}$

This is only the cost PER HOUR.

find the $\boxed{\text{total cost}}$ of the journey.

first find the number of hours of the journey.

$$S = \frac{D}{T}$$

$$V = \frac{200}{T}$$

$$\boxed{T = \frac{200}{V}}$$

$\therefore$ Total Cost

$$C = \underbrace{\left(V^2 + \frac{4000}{V}\right)}_{\text{cost per hour}} \times \underbrace{\frac{200}{V}}_{\text{number of hours}}$$

$$C = 200V + \frac{800\,000}{V^2}$$

$$\therefore \frac{dC}{dV} = +200 - \frac{1600000}{V^3} = 0 \quad \text{at SP's.}$$

$\times V^3$

$$200V^3 - 1600000 = 0$$

$$V^3 = \frac{16\,000\cancel{0}\cancel{0}}{2\cancel{0}\cancel{0}}$$

$$V^3 = 8000$$

$$V = 20 \text{ km/h.}$$

$$\frac{d^2C}{dV^2} = +\frac{6400000}{V^4}$$

when $V = 20$ $\quad \frac{d^2C}{dV^2} = +VE$ $\quad \therefore$ Min Cost value when $V = 20$ km/h.