Implicit 1 : Answers

1) $x^2+y^2 = 25$

Explicit

$y^2 = 25-x^2$

(*) $y = \sqrt{25-x^2}$

$y = (25-x^2)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(25-x^2)^{-1/2} \times (-2x)$

$\frac{dy}{dx} = \frac{-x}{\sqrt{25-x^2}}$

Implicit

$x^2+y^2 = 25$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$

Check implicit answer $-\frac{x}{y} = \frac{-x}{\sqrt{25-x^2}}$

because $y = \sqrt{25-x^2}$ from (*)

QED.

2) $3x^2+y^2 = 18$

Explicit

$y^2 = 18-3x^2$

(*) $y = (18-3x^2)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(18-3x^2)^{-1/2} \times (-6x)$

$= \frac{-3x}{\sqrt{18-3x^2}}$

Implicit

$3x^2+y^2 = 18$

$6x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -6x$

$\frac{dy}{dx} = \frac{-6x}{2y} = \frac{-3x}{y}$

From (*) $y = \sqrt{18-3x^2}$

$\therefore$ Implicit $-\frac{3x}{y} = \frac{-3x}{\sqrt{18-3x^2}}$

QED.

3) $2x^2 + 5y^2 = 20$

Explicit

$$5y^2 = 20 - 2x^2$$

$$y^2 = \frac{20 - 2x^2}{5}$$

$$y^2 = 4 - \frac{2}{5}x^2$$

(*) $$y = \sqrt{4 - \frac{2}{5}x^2} = \left(4 - \frac{2}{5}x^2\right)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(4 - \frac{2}{5}x^2\right)^{-1/2} \times \left(\frac{-4}{5}x\right)$$

$$= \frac{\frac{-4}{10}x}{\sqrt{4 - \frac{2}{5}x^2}} = \frac{-2x}{5\sqrt{4 - \frac{2}{5}x^2}}$$

Implicit

$$2x^2 + 5y^2 = 20$$

$$4x + 10y\frac{dy}{dx} = 0$$

$$10y\frac{dy}{dx} = -4x$$

$$\frac{dy}{dx} = \frac{-4x}{10y} = \frac{-2x}{5y}$$

from (*) $$= \frac{-2x}{5\sqrt{4 - \frac{2}{5}x^2}}$$

QED.

4) $5x^2 - 2y^2 = 8$

Explicit

$5x^2 - 8 = 2y^2$

$\frac{5}{2}x^2 - 4 = y^2$

(*) $y = \sqrt{\frac{5}{2}x^2 - 4} = \left(\frac{5}{2}x^2 - 4\right)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{5}{2}x^2 - 4\right)^{-1/2} \times (5x)$

$= \frac{5x}{2\sqrt{\frac{5}{2}x^2 - 4}}$

Implicit

$5x^2 - 2y^2 = 8$

$10x - 4y\frac{dy}{dx} = 0$

$10x = 4y\frac{dy}{dx}$

$\frac{10x}{4y} = \frac{dy}{dx}$

$\frac{5x}{2y} = \frac{dy}{dx}$

From (*)

$\frac{5x}{2\sqrt{\frac{5}{2}x^2 - 4}} = \frac{dy}{dx}$

QED.

5) $xy = 5$

Explicit

(*) $y = \frac{5}{x}$

$y = 5x^{-1}$

$\frac{dy}{dx} = -5x^{-2}$

$\frac{dy}{dx} = \frac{-5}{x^2}$

Implicit

$xy = 5$

$x\frac{dy}{dx} + y = 0$

$x\frac{dy}{dx} = -y$

$\frac{dy}{dx} = \frac{-y}{x}$

Product Rule

$u = x \quad v = y$

$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \frac{dy}{dx}$

$u\frac{dv}{dx} + v\frac{du}{dx}$

$= x\frac{dy}{dx} + y(1)$

from (*) $\frac{dy}{dx} = \frac{-5/x}{x}$

$= \frac{-5}{x^2}$

QED.

6) $x^2y = 7$

Explicit

$y = \frac{7}{x^2}$

$y = 7x^{-2}$

$\frac{dy}{dx} = -14x^{-3}$

$= \frac{-14}{x^3}$

Implicit

$x^2y = 7$

$x^2\frac{dy}{dx} + 2xy = 0$

$x^2\frac{dy}{dx} = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{x^2}$

$= \frac{-2y}{x}$

Product Rule

$u = x^2 \quad v = y$

$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = \frac{dy}{dx}$

$u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^2\frac{dy}{dx} + y(2x)$

from (*) $\frac{dy}{dx} = \frac{-2\left(\frac{7}{x^2}\right)}{x}$

$= \frac{-14}{x^3}$

QED.

7) $ye^x = 4$

Explicit

(*) $y = \frac{4}{e^x}$

$y = 4e^{-x}$

$\frac{dy}{dx} = -4e^{-x}$

$\frac{dy}{dx} = -\frac{4}{e^x}$

Implicit

$ye^x = 4$

$ye^x + e^x\frac{dy}{dx} = 0$

$e^x\frac{dy}{dx} = -ye^x$

$\frac{dy}{dx} = \frac{-ye^x}{e^x}$

$\frac{dy}{dx} = -y$

(*) $\frac{dy}{dx} = -\frac{4}{e^x}$

Product Rule

$u = y \quad v = e^x$

$\frac{du}{dx} = \frac{dy}{dx} \quad \frac{dv}{dx} = e^x$

$v\frac{dv}{dx} + v\frac{du}{dx}$

$= ye^x + e^x\frac{dy}{dx}$

QED.

8) $y^2x = 1$

Explicit

$y^2 = \frac{1}{x}$

$y^2 = x^{-1}$

$y = (x^{-1})^{1/2}$

(*) $y = x^{-1/2} = \frac{1}{\sqrt{x}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-3/2}$

$\frac{dy}{dx} = -\frac{1}{2\sqrt{x^3}}$

Implicit

$y^2x = 1$

$y^2 + 2xy\frac{dy}{dx} = 0$

$2xy\frac{dy}{dx} = -y^2$

$\frac{dy}{dx} = \frac{-y^2}{2xy}$

$\frac{dy}{dx} = -\frac{y}{2x}$

from (*) $\Rightarrow \frac{dy}{dx} = \frac{-\frac{1}{\sqrt{x}}}{2x} = -\frac{1}{2\sqrt{x}x}$

$= -\frac{1}{2x^{3/2}}$

$= -\frac{1}{2\sqrt{x^3}}$

Product Rule

$u = y^2 \quad v = x$

$\frac{du}{dx} = 2y\frac{dy}{dx} \quad \frac{dv}{dx} = 1$

$u\frac{dv}{dx} + v\frac{du}{dx}$

$= y^2(1) + x2y\frac{dy}{dx}$

QED.

9) $x^2y + y = 4$

Explicit

Common factor $y(x^2+1) = 4$

(*) $y = \frac{4}{(x^2+1)}$

$y = 4(x^2+1)^{-1}$

$\frac{dy}{dx} = -4(x^2+1)^{-2} \times 2x$

$\frac{dy}{dx} = \frac{-8x}{(x^2+1)^2}$

Implicit

$x^2y + y = 4$

$x^2\frac{dy}{dx} + 2xy + \frac{dy}{dx} = 0$

$\frac{dy}{dx}(x^2+1) = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$

Product Rule x^2y

$u = x^2 \qquad v = y$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \frac{dy}{dx}$

$u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^2\frac{dy}{dx} + y(2x)$

(*) $\frac{dy}{dx} = \frac{-2x\,\frac{4}{(x^2+1)}}{(x^2+1)}$

$= \frac{-8x}{(x^2+1)^2}$

QED.