

Task 11 : Answers

1) $y = x(x^2 - 6)$

$$u = x \quad v = x^2 - 6$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = 2x$$

$$\begin{aligned} \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= x(2x) + 1(x^2 - 6) \\ &= 2x^2 + x^2 - 6 \\ &= 3x^2 - 6 \\ &= 3(x^2 - 2) \end{aligned}$$

CHECK $y = x(x^2 - 6)$
expand $y = x^3 - 6x$
 $\frac{dy}{dx} = 3x^2 - 6$
 $= 3(x^2 - 2)$

2) $y = x^2(x^2 + 3x)$

$$u = x^2 \quad v = x^2 + 3x$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = 2x + 3$$

$$\begin{aligned} \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= x^2(2x + 3) + 2x(x^2 + 3x) \\ &= 2x^3 + 3x^2 + 2x^3 + 6x^2 \\ &= 4x^3 + 9x^2 \\ &= x^2(4x + 9) \end{aligned}$$

CHECK $y = x^2(x^2 + 3x)$
 $y = x^4 + 3x^3$
 $\frac{dy}{dx} = 4x^3 + 9x^2$
 $= x^2(4x + 9)$

$$3) f(x) = x^3(3x-7)$$

$$u = x^3 \quad v = 3x-7$$

$$\frac{du}{dx} = 3x^2 \quad \frac{dv}{dx} = 3$$

$$\begin{aligned} f'(x) &= \frac{u dv}{dx} + \frac{v du}{dx} \\ &= 3x^3 + (3x-7)3x^2 \\ &= 3x^3 + 9x^3 - 21x^2 \\ &= 12x^3 - 21x^2 \\ &= 3x^2(4x-7) \end{aligned}$$

CHECK

$$f(x) = x^3(3x-7)$$

$$f(x) = 3x^4 - 7x^3$$

$$f'(x) = 12x^3 - 21x^2$$

$$= 3x^2(4x-7)$$

4) Here we first say

$$\text{Let } y = 2x(x^2+4)$$

$$u = 2x \quad v = x^2+4$$

$$\frac{du}{dx} = 2 \quad \frac{dv}{dx} = 2x$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{u dv}{dx} + \frac{v du}{dx} \\ &= 2x(2x) + 2(x^2+4) \\ &= 4x^2 + 2x^2 + 8 \\ &= 6x^2 + 8 \\ &= 2(3x^2+4) \end{aligned}$$

CHECK

$$y = 2x(x^2+4)$$

$$y = 2x^3 + 8x$$

$$\frac{dy}{dx} = 6x^2 + 8$$

$$= 2(3x^2+4)$$

$$5) \quad y = 2x^2(3x+2)$$

$$u = 2x^2 \quad v = 3x+2$$

$$\frac{du}{dx} = 4x \quad \frac{dv}{dx} = 3$$

$$\begin{aligned} \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= 2x^2(3) + 4x(3x+2) \\ &= 6x^2 + 12x^2 + 8x \\ &= 18x^2 + 8x \\ &= 2x(9x+4) \end{aligned}$$

CHECK

$$y = 2x^2(3x+2)$$

$$y = 6x^3 + 4x^2$$

$$\frac{dy}{dx} = 18x^2 + 8x$$

$$= 2x(9x+4)$$

$$6) \quad f(x) = 2x^3(3x^2+5x)$$

$$u = 2x^3 \quad v = 3x^2+5x$$

$$\frac{du}{dx} = 6x^2 \quad \frac{dv}{dx} = 6x+5$$

$$\begin{aligned} f'(x) &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= 2x^3(6x+5) + 6x^2(3x^2+5x) \\ &= 12x^4 + 10x^3 + 18x^4 + 30x^3 \\ &= 30x^4 + 40x^3 \\ &= 10x^3(3x+4) \end{aligned}$$

CHECK

$$y = 2x^3(3x^2+5x)$$

$$y = 6x^5 + 10x^4$$

$$\frac{dy}{dx} = 30x^4 + 40x^3$$

$$= 10x^3(3x+4)$$

$$7) \quad y = (2x+1)(3x-2)$$

$$u = 2x+1 \quad v = 3x-2$$

$$\frac{du}{dx} = 2 \quad \frac{dv}{dx} = 3$$

$$\begin{aligned} \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= 3(2x+1) + 2(3x-2) \\ &= 6x+3 + 6x-4 \\ &= 12x-1 \end{aligned}$$

CHECK

$$\begin{aligned} y &= (2x+1)(3x-2) \\ y &= 6x^2 - 4x + 3x - 2 \\ y &= 6x^2 - x - 2 \\ \frac{dy}{dx} &= 12x - 1 \end{aligned}$$

$$\begin{aligned} 8) \quad y &= (x^2+3)(x^2-3) \\ u &= x^2+3 \quad v = x^2-3 \\ \frac{du}{dx} &= 2x \quad \frac{dv}{dx} = 2x \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= 2x(x^2+3) + 2x(x^2-3) \\ &= 2x^3+6x + 2x^3-6x \\ &= 4x^3 \end{aligned}$$

CHECK

$$\begin{aligned} y &= (x^2+3)(x^2-3) \\ y &= x^4 + 3x^2 - 3x^2 - 9 \\ y &= x^4 - 9 \\ \frac{dy}{dx} &= 4x^3 \end{aligned}$$

$$9) f(x) = (2x^2 + x)(x^2 - 3x)$$

$$u = (2x^2 + x) \quad v = x^2 - 3x$$

$$\frac{du}{dx} = 4x + 1$$

$$\frac{dv}{dx} = 2x - 3$$

$$f'(x) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$= (2x^2 + x)(2x - 3) + (x^2 - 3x)(4x + 1)$$

$$= 4x^3 - 6x^2 + 2x^2 - 3x + 4x^3 + x^2 - 12x^2 - 3x$$

$$= 8x^3 - 15x^2 - 6x$$

$$= x(8x^2 - 15x - 6)$$

CHECK

$$f(x) = (2x^2 + x)(x^2 - 3x)$$

$$f(x) = 2x^4 - 6x^3 + x^3 - 3x^2 = 2x^4 - 5x^3 - 3x^2$$

$$f'(x) = 8x^3 - 15x^2 - 6x$$

$$= x(8x^2 - 15x - 6)$$

$$10) \text{ Let } y = (3x + x^2)(4 - x^2)$$

$$u = 3x + x^2 \quad v = 4 - x^2$$

$$\frac{du}{dx} = 3 + 2x$$

$$\frac{dv}{dx} = -2x$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$= (3x + x^2)(-2x) + (4 - x^2)(3 + 2x)$$

$$= -6x^2 - 2x^3 + [12 + 8x - 3x^2 - 2x^3]$$

$$= -6x^2 - 2x^3 + 12 + 8x - 3x^2 - 2x^3$$

$$= -4x^3 - 9x^2 + 8x + 12$$

CHECK

$$y = (3x + x^2)(4 - x^2)$$

$$y = 12x - 3x^3 + 4x^2 - x^4$$

$$\frac{dy}{dx} = 12 - 9x^2 + 8x - 4x^3$$

$$= -4x^3 - 9x^2 + 8x + 12$$