Task 13 : Product Rule + Standard Derivatives : Answers

1) $y = x^3 \cos x$

$u = x^3 \qquad v = \cos x$

$\frac{du}{dx} = 3x^2 \qquad \frac{dv}{dx} = -\sin x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= x^3(-\sin x) + 3x^2\cos x$$

$$= x^2(3\cos x - x\sin x)$$

2) $y = 2x^2 e^x$

$u = 2x^2 \qquad v = e^x$

$\frac{du}{dx} = 4x \qquad \frac{dv}{dx} = e^x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= 2x^2 e^x + 4x e^x$$

$$= 2x e^x (x + 2)$$

3) $f(x) = x^3 \ln x$

$u = x^3 \qquad v = \ln x$

$\frac{du}{dx} = 3x^2 \qquad \frac{dv}{dx} = \frac{1}{x}$

$$f'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= x^3\left(\frac{1}{x}\right) + 3x^2 \ln x$$

$$= x^2 + 3x^2 \ln x$$

$$= x^2(1 + 3\ln x)$$

4) $f(x) = 3x^2 \sin x$

$u = 3x^2 \qquad v = \sin x$

$\frac{du}{dx} = 6x \qquad \frac{dv}{dx} = \cos x$

$f'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$= 3x^2\cos x + 6x\sin x$

$= 3x(x\cos x + 2\sin x)$

5) $f(x) = x\tan x$

$u = x \qquad v = \tan x$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \sec^2 x$

$f'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x\sec^2 x + (1)\tan x$

$= x\sec^2 x + \tan x$

6) Let $y = e^x \cos x$

$u = e^x \qquad v = \cos x$

$\frac{du}{dx} = e^x \qquad \frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= e^x(-\sin x) + e^x\cos x$

$= e^x(\cos x - \sin x)$

7) Let $y = e^x \ln x$

$u = e^x \qquad v = \ln x$

$\frac{du}{dx} = e^x \qquad \frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= e^x\left(\frac{1}{x}\right) + e^x\ln x$

$= e^x\left(\frac{1}{x} + \ln x\right)$

carry on if you want to

$e^x\left(\frac{1}{x} + \frac{x\ln x}{x}\right)$

$= e^x\left(\frac{1 + x\ln x}{x}\right)$

$= \frac{e^x(1 + x\ln x)}{x}$

8) Let $y = (2+3x)e^x$

$u = 2+3x \qquad v = e^x$

$\frac{du}{dx} = 3 \qquad \frac{dv}{dx} = e^x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= (2+3x)e^x + 3e^x$$

$$= e^x\left[(2+3x) + 3\right]$$

$$= e^x(5+3x)$$

9) $y = x^2 \ln|x+2|$

$u = x^2 \qquad v = \ln|x+2|$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \frac{1}{x+2}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= x^2\left(\frac{1}{x+2}\right) + 2x\ln|x+2|$$

$$= x\left[\frac{x}{x+2} + 2\ln|x+2|\right]$$ ← This gets full marks

BUT You could do this for a neater answer

$$= x\left[\frac{x}{x+2} + \frac{2(x+2)\ln|x+2|}{x+2}\right]$$

$$= x\left[\frac{x + 2(x+2)\ln|x+2|}{x+2}\right]$$

$$= \frac{x\left[x + 2(x+2)\ln|x+2|\right]}{x+2}$$

It is neater because it is a single fraction !!

10) $f(x) = (x^2-1)\sin x$

$u = x^2-1 \qquad v = \sin x$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \cos x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (x^2-1)\cos x + 2x\sin x$

There is no common factor!