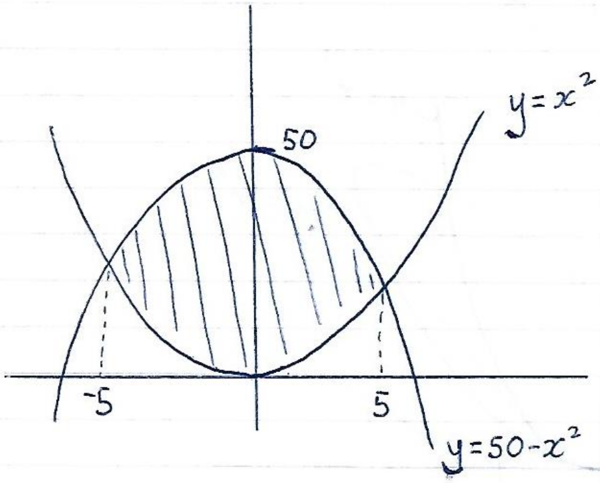
Areas Between 2 Curves : Answers

1)



Find intersection point x values.

$$x^2 = 50 - x^2$$
$$2x^2 = 50$$
$$x^2 = 25$$
$$x = \pm 5$$

$$\therefore \text{Area} = \int_{-5}^{5} 50 - x^2\,dx - \int_{-5}^{5} x^2\,dx$$

$$= \int_{-5}^{5} 50 - 2x^2\,dx$$

$$= \left[50x - \frac{2x^3}{3}\right]_{-5}^{5}$$

$$= \left(250 - \frac{250}{3}\right) - \left(-250 + \frac{250}{3}\right)$$

$$= 250 + 250 - \frac{250}{3} - \frac{250}{3}$$

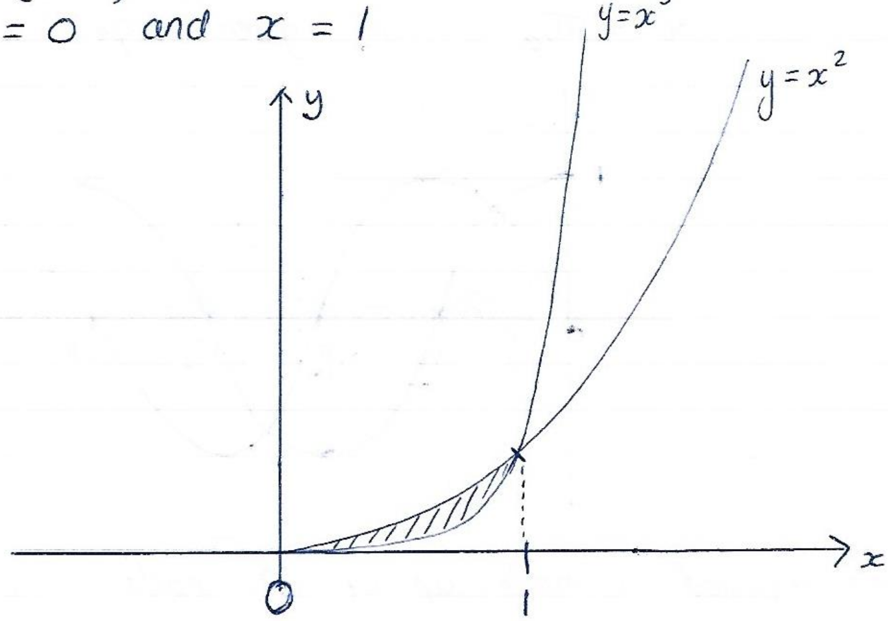
$$= 500 - \frac{500}{3}$$

$$= \frac{1000}{3} \text{ units}^2$$

2) $y = x^3$ and $y = x^2$

x intersection points

$$x^3 = x^2$$
$$x^3 - x^2 = 0$$
$$x^2(x-1) = 0$$
$$x = 0 \quad \text{and} \quad x = 1$$



* Notice between 0 and 1 x^2 is above x^3 graph.

for values of x above 1 x^3 is above x^2

$$\text{Area shaded} = \int_0^1 x^2 - x^3 \, dx$$
$$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$
$$= \left(\frac{1}{3} - \frac{1}{4}\right) - (0 - 0)$$
$$= \frac{1}{12} \text{ units}^2$$

3) $y = x^3$ and $y = x^2$

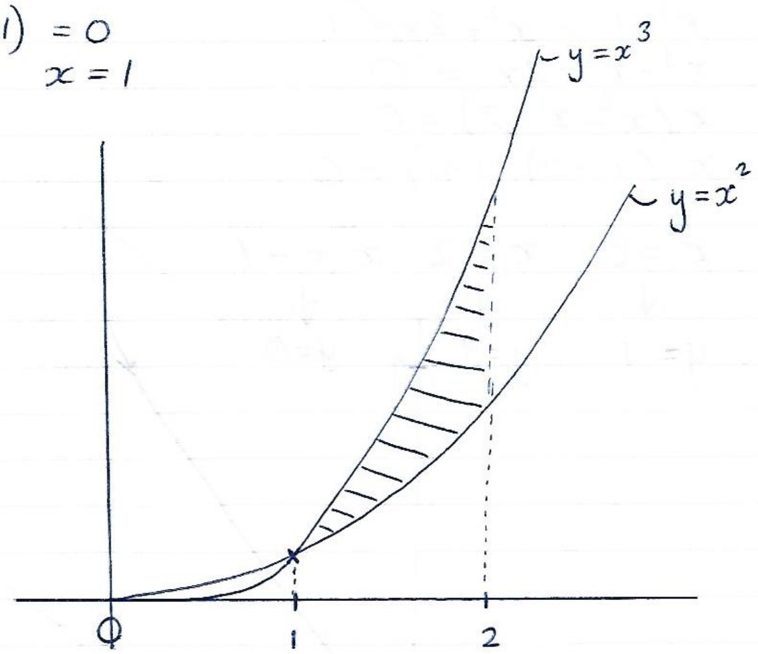
x intersection points

$x^3 = x^2$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0 \quad x = 1$



* Between $x = 1$ and $x = 2$ x^3 is above x^2 graph

$$\text{Area} = \int_1^2 x^3\,dx - \int_1^2 x^2\,dx$$

$$= \int_1^2 x^3 - x^2\,dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^2$$

$$= \left(4 - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)$$

$$= 4 - \frac{8}{3} - \frac{1}{4} + \frac{1}{3}$$

$$= \frac{48}{12} - \frac{32}{12} - \frac{3}{12} + \frac{4}{12}$$

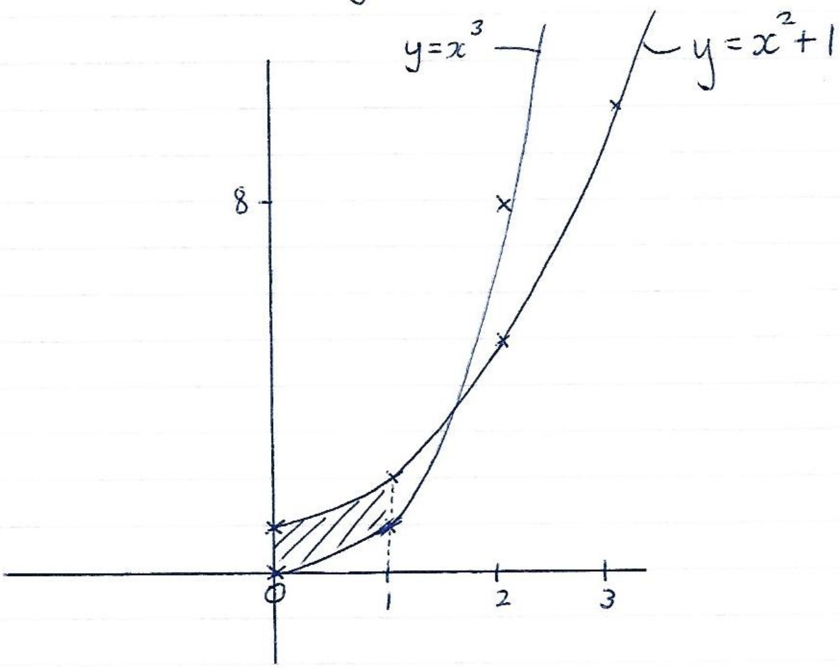
$$= \frac{17}{12} \text{ units}^2$$

4) $y = x^2 + 1$ and $y = x^3$

A decent sketch is needed.

x	0	1	2	3
y	1	2	5	10

x	0	1	2	3
y	0	1	8	27



$$\text{Area shaded} = \int_0^1 x^2+1 - x^3 \, dx$$

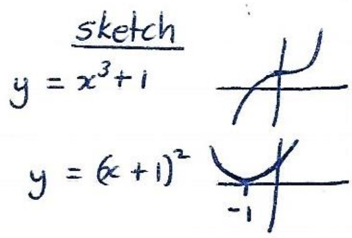
$$= \left[\frac{x^3}{3} + x - \frac{x^4}{4}\right]_0^1$$

$$= \left(\frac{1}{3} + 1 - \frac{1}{4}\right) - 0$$

$$= \frac{13}{12} \text{ units}^2$$

5) $y = x^3+1$ and $y = (x+1)^2$

x value intersections



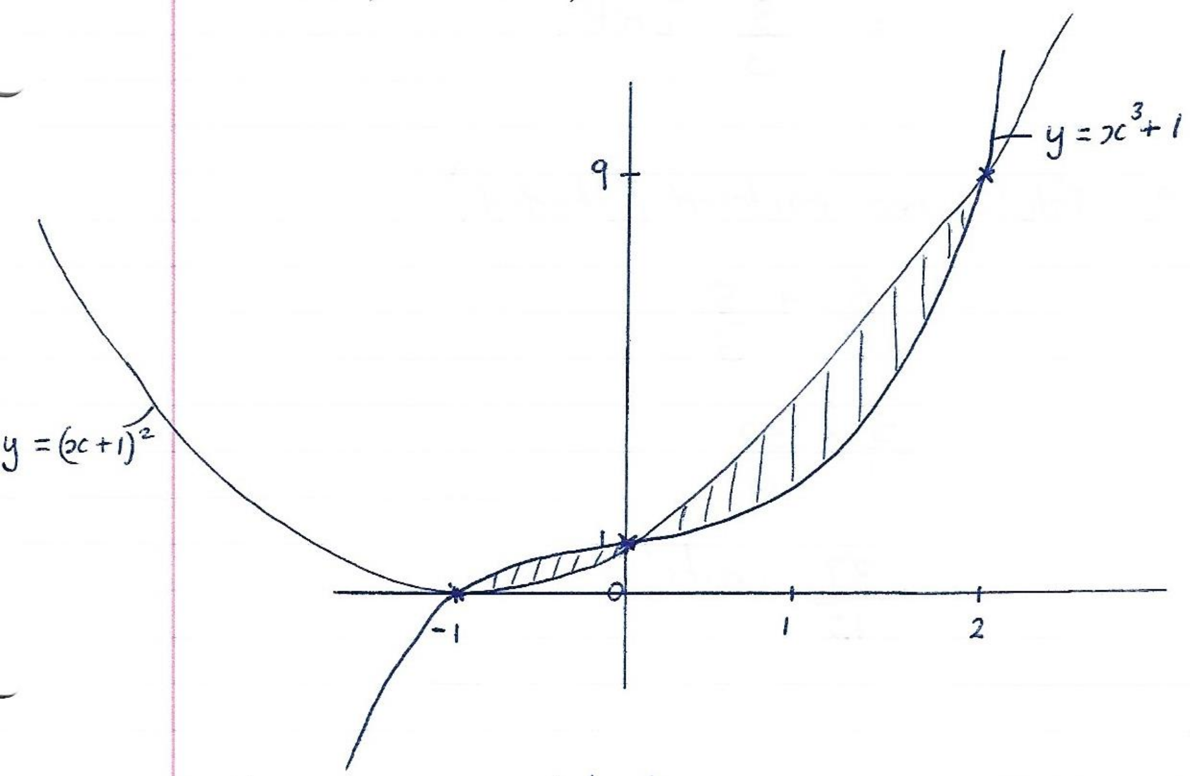
$$x^3+1 = (x+1)^2$$
$$x^3+1 = x^2+2x+1$$
$$x^3-x^2-2x = 0$$
$$x(x^2-x-2) = 0$$
$$x(x-2)(x+1) = 0$$

$x = 0$ $\quad x = 2$ $\quad x = -1$

$\downarrow$ $\quad\downarrow$ $\quad\downarrow$

$y = 1$ $\quad y = 9$ $\quad y = 0$

$(0,1)$ $\quad (2,9)$ $\quad (-1,0)$



There are 2 distinct regions.

$$\text{First area} = \int_{-1}^{0} x^3+1 - (x+1)^2\,dx$$

$$= \int_{-1}^{0} x^3+1-(x^2+2x+1)\,dx = \int_{-1}^{0} x^3+1-x^2-2x-1\,dx$$

$$= \int_{-1}^{0} x^3-x^2-2x\,dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{2x^2}{2}\right]_{-1}^{0}$$

$$= (0) - \left(\frac{1}{4} + \frac{1}{3} - 1\right)$$

$$= 0 - \frac{1}{4} - \frac{1}{3} + 1 = \frac{5}{12} \text{ units}^2$$

Second area $= \int_0^2 (x+1)^2 - (x^3+1)\, dx$

$$= \int_0^2 x^2 + 2x + 1 - x^3 - 1 \quad dx$$

$$= \int_0^2 -x^3 + x^2 + 2x \;\, dx$$

$$= \left[\frac{-x^4}{4} + \frac{x^3}{3} + x^2 \right]_0^2$$

$$= \left[\left(-4 + \frac{8}{3} + 4 \right) - 0 \right]$$

$$= \frac{8}{3} \text{ units}^2$$

$\therefore$ Total area enclosed (shaded)

$$= \frac{5}{12} + \frac{8}{3}$$

$$= \frac{5}{12} + \frac{32}{12}$$

$$= \frac{37}{12} \text{ units}^2$$