# Cubics 3 : Answers : NO ACCOUNT TAKEN OF MAX OR MIN POINT POSITION !!!

* CRUDE SKETCHES ONLY.

1) a) $y = x^3 - 7x - 6$

Crosses $y$ axis when $x = 0$

$y = 0 - 0 - 6$

$y = -6$

Crosses $x$ axis when $y = 0$

$0 = x^3 - 7x - 6$

RHS $f(1) = 1 - 7 - 6 \neq 0$

$f(-1) = -1 + 7 - 6 = 0$

$\therefore x + 1$ is a factor

Coeffts method

$x^3 - 7x - 6 = (x+1)(ax^2 + bx + c)$

| Compare $x^3$ | Compare consts | Compare $x^2$ |
|---|---|---|
| $1 = a$ | $-6 = c$ | $0 = a + b$ |
| | | $0 = 1 + b$ |
| | | $-1 = b$ |

$\therefore$ Curve crosses $x$ axis

$0 = x^3 - 7x - 6$

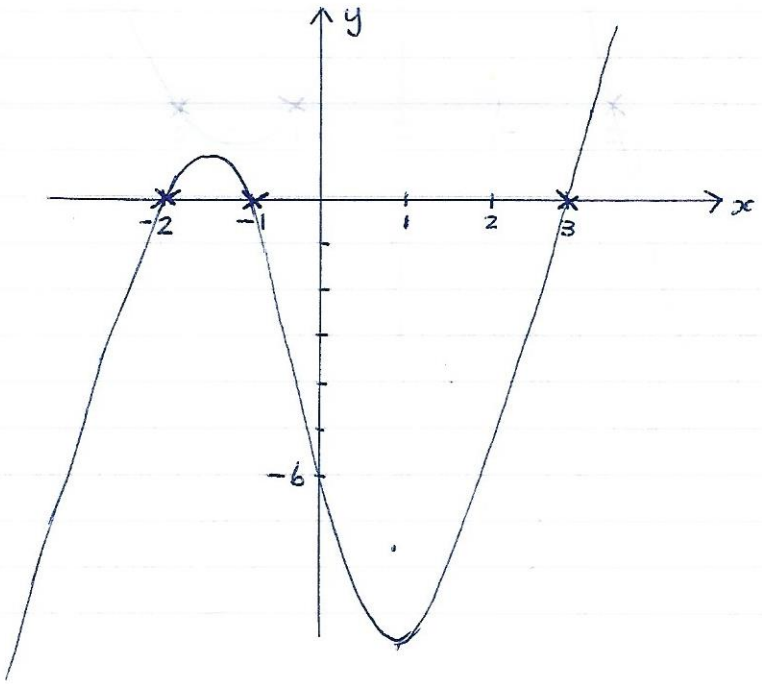
$0 = (x+1)(x^2 - x - 6)$

$0 = (x+1)(x-3)(x+2)$

$\therefore$ Crosses $x$ axis at

$x = -1 \quad x = 3 \quad x = -2$

Sketch




* $y = x^3 - 7x - 6$ is +ve $x^3$

b) $y = x^3 - x^2 - 4x + 4$

Crosses y axis, $x = 0$

$y = 0 - 0 - 0 + 4$

$y = 4$

Crosses x axis, $y = 0$

$0 = x^3 - x^2 - 4x + 4$

RHS

$f(1) = 1 - 1 - 4 + 4 = 0$

$\therefore (x-1)$ is a factor

Coeffts method

$x^3 - x^2 - 4x + 4 = (x-1)(ax^2 + bx + c)$

Compare $x^3$: $1 = a$

Compare consts: $4 = -c$, $-4 = c$

Compare $x^2$: $-1 = -a + b$, $-1 = -1 + b$, $0 = b$

$\therefore$ Crosses x axis

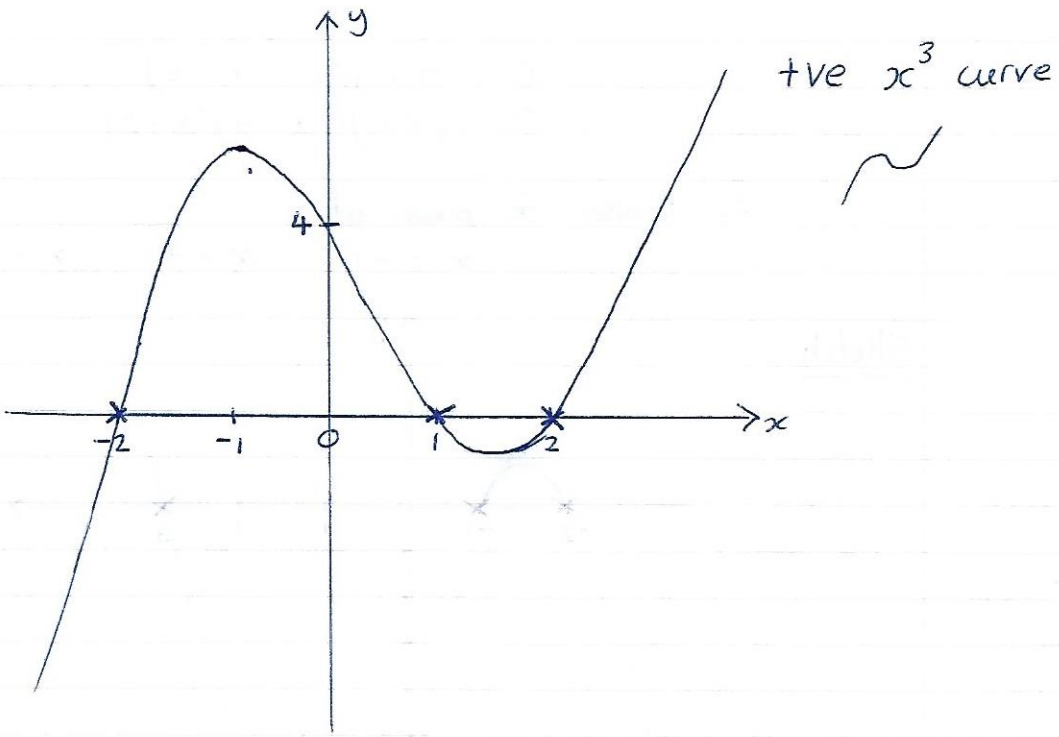
$0 = x^3 - x^2 - 4x + 4$

$0 = (x-1)(x^2 - 4)$

$0 = (x-1)(x+2)(x-2)$

$x = 1 \quad x = -2 \quad x = 2$

Sketch



c) $y = x^3 + 4x^2 - x - 4$

Crosses y axis, $x = 0$

$y = 0 + 0 - 0 - 4$

$y = -4$

Crosses x axis, $y = 0$

$0 = x^3 + 4x^2 - x - 4$

RHS $f(1) = 1 + 4 - 1 - 4 = 0$

$\therefore (x-1)$ is a factor

Coeffts method

$x^3 + 4x^2 - x - 4 = (x-1)(ax^2 + bx + c)$

Compare $x^3$: $1 = a$

Compare consts: $-4 = -c$, $4 = c$

Compare $x^2$: $4 = -a + b$, $4 = -1 + b$, $5 = b$

$\therefore$ Crosses x axis

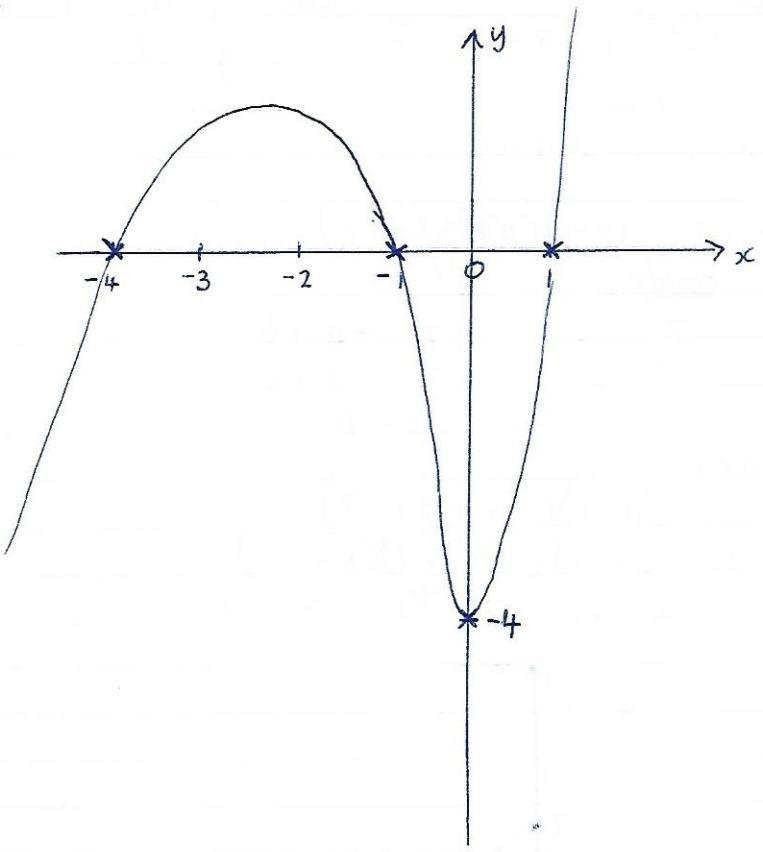
$0 = x^3 + 4x^2 - x - 4$

$0 = (x-1)(x^2 + 5x + 4)$

$0 = (x-1)(x+4)(x+1)$

$x = 1 \quad x = -4 \quad x = -1$

Sketch.




+ $x^3$ graph

d) $y = x^3 - 9x$

Crosses $x$ axis $y = 0$

$0 = x^3 - 9x$

$0 = (x^2 - 9)x$

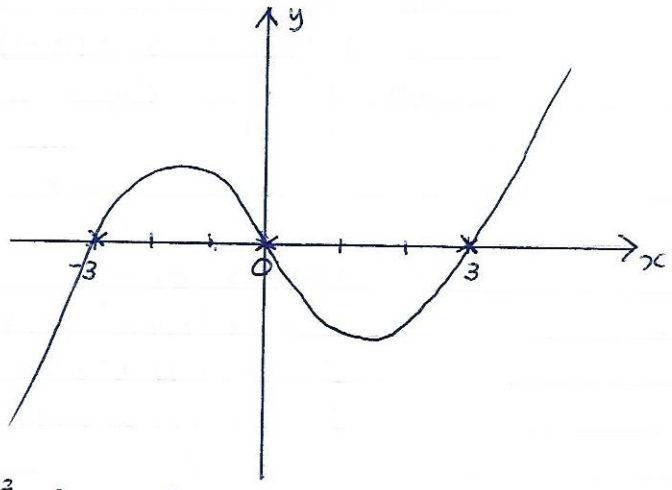
$0 = x(x+3)(x-3)$

$x = 0 \quad x = -3 \quad x = 3$

Crosses $y$ axis $x = 0$

$y = 0 - 0$

$y = 0$




+ve $x^3$

e) $y = 2x^3 - 7x^2 - 2x + 7$

Crosses $x$ axis $y = 0$

$0 = 2x^3 - 7x^2 - 2x + 7$

$f(1) = 2 - 7 - 2 + 7 = 0$

$\therefore (x-1)$ is a factor.

$2x^3 - 7x^2 - 2x + 7 = (x-1)(ax^2 + bx + c)$

$x^3$: $2 = a$

consts: $7 = -c$, $-7 = c$

$x^2$: $-7 = -a + b$, $-7 = -2 + b$, $-5 = b$

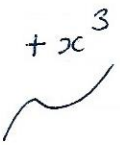
$\therefore$ Crosses $x$ axis

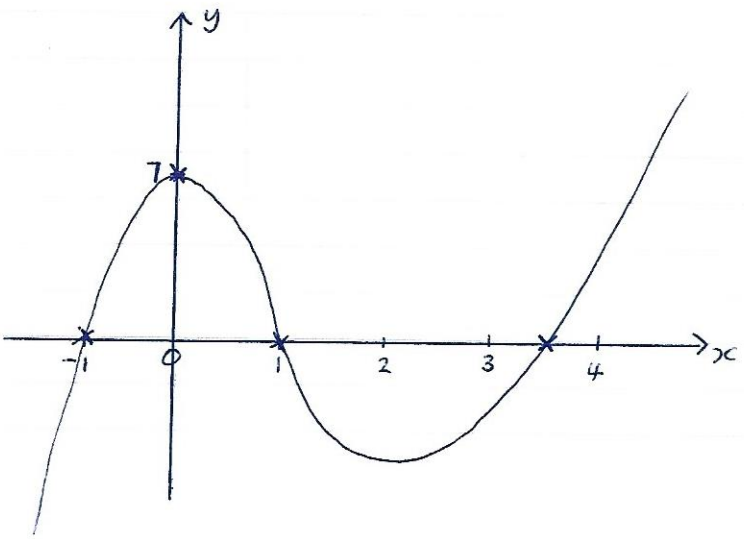
$0 = (x-1)(2x^2 - 5x - 7)$

$0 = (x-1)(2x-7)(x+1)$

$x = 1 \quad x = 7/2 \quad x = -1$

+ $x^3$





f) $y = 2x^3 - 11x^2 + 5x + 18$

Crosses $x$ axis $y = 0$

$0 = 2x^3 - 11x^2 + 5x + 18$

Crosses $y$ axis $x = 0$

$y = 18$

RHS $f(1) = 2 - 11 + 5 + 18 \neq 0$

$f(-1) = -2 - 11 - 5 + 18 = 0$

$\therefore (x+1)$ is a factor

RHS $2x^3 - 11x^2 + 5x + 18 = (x+1)(ax^2 + bx + c)$

$x^3$ | consts | $x^2$

$2 = a$ | $18 = c$ | $-11 = a + b$

$-11 = 2 + b$

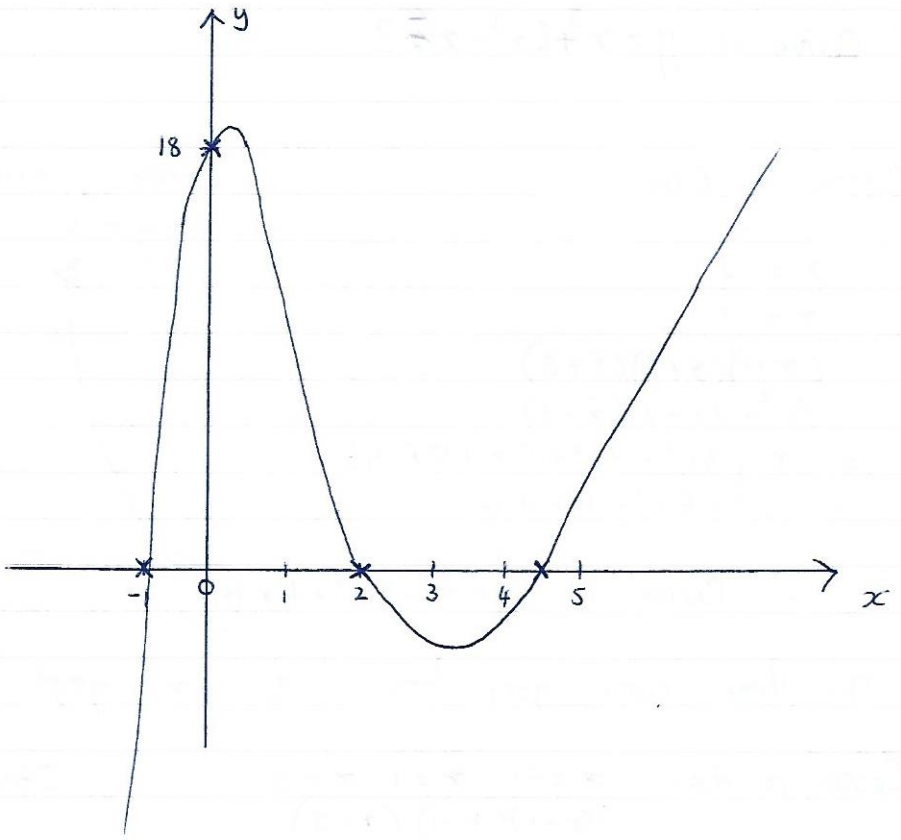
$-13 = b$

$\therefore$ Crosses $x$ axis

$(x+1)(2x^2 - 13x + 18) = 0$

$(x+1)(2x - 9)(x - 2) = 0$

$x = -1 \quad x = 9/2 \quad x = 2$



2) a) Crosses $x$ axis

$x = -1$

$x = 1$

$x = 3$

Crosses $y$ axis, $x = 0$

$y = 3$

$\therefore (x-3)(x-1)(x+1)$

$= (x-3)(x^2-1)$

$= x^3 - x - 3x^2 + 3$

$= x^3 - 3x^2 - x + 3$

$\therefore$ Curve is

$y = x^3 - 3x^2 - x + 3$

b) Crosses $x$ axis

$x = -2$

$x = -1$

$x = 1$

$\therefore (x-1)(x+1)(x+2)$

$= (x^2-1)(x+2)$

$= x^3 + 2x^2 - x - 2$

Crosses $y$ axis, $x = 0$

$y = -2$

$\therefore$ Curve is $y = x^3 + 2x^2 - x - 2$

c) Crosses $x$ axis

$x = -3$

$x = -2$

$x = -1$

$\therefore (x+1)(x+2)(x+3)$

$= (x^2 + 3x + 2)(x+3)$

$= x^3 + 3x^2 + 3x^2 + 9x + 2x + 6$

$= x^3 + 6x^2 + 11x + 6$

Crosses $y$ axis, $x = 0$

$y = 6$

$\therefore$ Curve is $y = x^3 + 6x^2 + 11x + 6$

d) * This time curve goes $\backsim$ $\therefore$ $-x^3$ graph !!

Crosses $x$ axis $x = -1$ $x = 1$ $x = 3$

$\therefore (x+1)(x-1)(x-3)$

$= (x^2-1)(x-3)$

$= x^3 - 3x^2 - x + 3$

BUT We Must have $-x^3$

$\therefore y = -x^3 + 3x^2 + x - 3$

Crosses $y$ axis, $x = 0$

$y = -3$