Sketching Curves : Answers

1) $y = x^3(x-1)$

$y = x^4 - x^3$

$\frac{dy}{dx} = 4x^3 - 3x^2 = 0$ at SP's.

$x^2(4x-3) = 0$

either $x^2 = 0$ or $4x - 3 = 0$

$x = 0$ $\qquad$ $x = \frac{3}{4}$

$y = 0^4 - 0^3$ $\qquad$ $y = \left(\frac{3}{4}\right)^4 - \left(\frac{3}{4}\right)^3$

$y = 0$ $\qquad$ $y = \frac{81}{256} - \frac{27}{64}$

$y = \frac{-27}{256}$

$(0,0)$ $\qquad$ $\left(\frac{3}{4}, \frac{-27}{256}\right)$

<u>New</u> $\frac{d^2y}{dx^2} = 12x^2 - 6x$

At $(0,0)$

$\frac{d^2y}{dx^2} = 0$ AMBIGUOUS

$\left(\frac{3}{4}, \frac{-27}{256}\right)$

$\frac{d^2y}{dx^2} = 12\left(\frac{3}{4}\right)^2 - 6\left(\frac{3}{4}\right)$

$= \frac{108}{16} - \frac{18}{4}$

$=$ +ve $\therefore$ LOCAL MIN

To see if we have inflection points

$\frac{d^2y}{dx^2} = 12x^2 - 6x = 0$ at inflection points

$2x^2 - x = 0$

$x(2x-1) = 0$

$x = 0$ or $x = \frac{1}{2}$

$x = 0$

$\downarrow$

$y = 0^4 - 0^3$

$y = 0$

$(0,0)$ inflection point but also SP !!

* We may check later if it does ⌒ or ⌓

Now $x = 1/2$

$\downarrow$

$y = \left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^3$

$y = \frac{1}{16} - \frac{1}{8}$

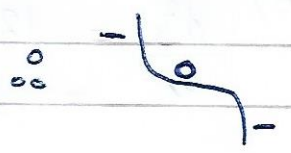
$y = -\frac{1}{16}$ $\left(\frac{1}{2}, -\frac{1}{16}\right)$ inflection point but NOT SP.

* Again not sure if ⌒ or ⌓

Now for $(0,0)$ which is SP and inflexion !

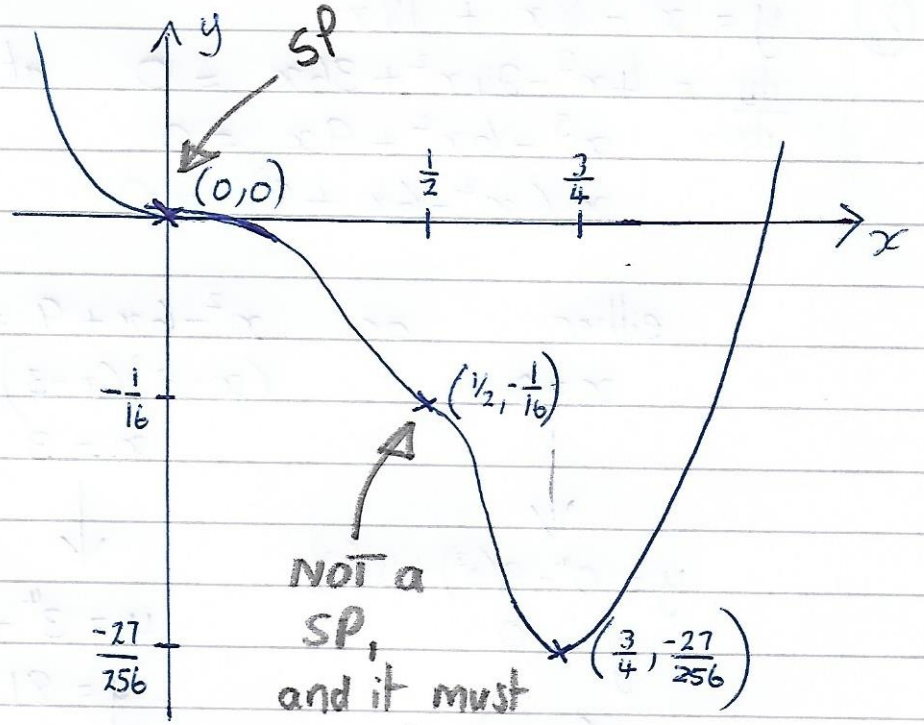
At $x = -0.1$ $\frac{dy}{dx} = 4(-0.1)^3 - 3(-0.1)^2$ $= -$ ve

$x = +0.1$ $\frac{dy}{dx} = 4(0.1)^3 - 3(0.1)^2$ $= -$ ve

∴ 

We may be able to do the sketch now without considering the gradients around $\left(\frac{1}{2}, -\frac{1}{16}\right)$

Sketch



NOT a SP, and it must go from other information to give a sketch that makes sense.

(2) $y = x^4 - 8x^3 + 18x^2$

$\frac{dy}{dx} = 4x^3 - 24x^2 + 36x = 0$ at SPs

$x^3 - 6x^2 + 9x = 0$

$x(x^2 - 6x + 9) = 0$

either $x = 0$ or $x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x = 3$

$y = 0^4 - 8(0^3) + 18(0^2)$

$y = 0$

$(0,0)$

$y = 3^4 - 8(3^3) + 18(3^2)$

$y = 81 - 216 + 162$

$y = +27$

$(3, +27)$

Nature?

$\frac{dy}{dx} = 4x^3 - 24x^2 + 36x$

$\frac{d^2y}{dx^2} = 12x^2 - 48x + 36$

(0,0) $x = 0$

$\frac{d^2y}{dx^2} = 0 - 0 + 36$

$\frac{d^2y}{dx^2} = +36$

(0,0) local min

$x = 3$

$\frac{d^2y}{dx^2} = 12(9) - 48(3) + 36$

$= 108 - 144 + 36$

$= 0$

(3,27) AMBIGUOUS

Now to find inflection points

$\frac{d^2y}{dx^2} = 12x^2 - 48x + 36 = 0$ at inflection

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$ or $x = 1$

$\downarrow$ $\downarrow$

(3,27) $y = 1 - 8 + 18 = 11$ (1, 11)

There are 2 inflection points

(3, 27) which is also a SP.

and (1, 11) which is NOT a SP.

Deciding which way the inflection points go.

(1, 11) ⌇ or ⌇

$x = +0.9$ $x = +1.1$

$\frac{dy}{dx} = 4x^3 - 24x^2 + 36x$

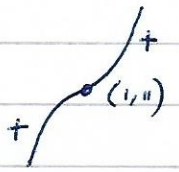
$= 4(0.9)^3 - 24(0.9)^2 + 36(0.9)$

$= +ve$

$\frac{dy}{dx} = 4(1.1)^3 - 24(1.1)^2 + 36(1.1)$

$= +ve$

∴ (1, 11) is



We can sketch now without repeating the above for (3, 27)

Sketch.