Integration By Substitution 3

1) June 2006
Use the substitution $u = 2\sin x + 3$ to evaluate

$$\int_0^{\pi/6} \frac{\cos x}{(2\sin x + 3)^2}\, dx$$

2) June 2009
Use the substitution $u = 2\cos x + 1$ to evaluate

$$\int_0^{\pi/3} \frac{\sin x}{\sqrt{2\cos x + 1}}\, dx$$

, showing the answer is $\sqrt{3} - \sqrt{2}$

3) June 2012
Use the substitution $u = 1 + 3\ln x$ to evaluate to 4 decimal places

$$\int_1^{e} \frac{1}{x(1 + 3\ln x)}\, dx$$

4) June 2014
Use the substitution $u = 10\cos x - 1$ to evaluate

$$\int_0^{\pi/3} \sqrt{10\cos x - 1}\, \sin x\, dx$$

5) June 2007

Use the substitution $x = 2\sin\theta$ to show that

$$\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4 - x^2}}\, dx = \int_0^{a} k\sin^2\theta\, d\theta$$

where a and k are values to be determined.
Hence evaluate $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4 - x^2}}\, dx$ to 3 decimal places

6) June 2008
Use the substitution $x = 3\sin\theta$ to show that

$$\int_{1.5}^{3} \sqrt{9 - x^2}\, dx = \int_a^{b} k\cos^2\theta\, d\theta$$

where a, b and k need to be found. Hence evaluate to 3 decimal places

$$\int_{1.5}^{3} \sqrt{9 - x^2}\, dx$$

Be Careful * 7) New Exam Specimen Paper

Use a suitable substitution to show that

$$\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = \frac{\pi}{12} - \frac{\sqrt{3}}{8}$$