Partial Fractions 2 : Answers

1)

a) $\dfrac{3x-2}{(x+2)(x-1)} = \dfrac{A}{(x+2)} + \dfrac{B}{(x-1)}$

$= \dfrac{A(x-1) + B(x+2)}{(x+2)(x-1)}$

$\therefore \quad 3x-2 = A(x-1) + B(x+2)$

Let $x = 1$

$3-2 = 0 + 3B$

$\dfrac{1}{3} = B$

Let $x = -2$

$-6-2 = -3A + 0$

$-8 = -3A$

$\dfrac{8}{3} = A$

$\therefore \quad \dfrac{3x-2}{(x+2)(x-1)} = \dfrac{8}{3(x+2)} - \dfrac{1}{3(x-1)}$

b) $\dfrac{-5}{(2x+1)(x-2)} = \dfrac{A}{(2x+1)} + \dfrac{B}{(x-2)}$

$= \dfrac{A(x-2) + B(2x+1)}{(2x+1)(x-2)}$

Let $x = 2$

$-5 = 0 + 5B$

$-1 = B$

Let $x = -\frac{1}{2}$

$-5 = \frac{-5}{2}A + 0$

$2 = A$

$\therefore \quad \dfrac{-5}{(2x+1)(x-2)} = \dfrac{2}{(2x+1)} - \dfrac{1}{(x-2)}$

c) $\frac{x+1}{x(3x-1)} = \frac{A}{x} + \frac{B}{(3x-1)}$

$= \frac{A(3x-1) + Bx}{x(3x-1)}$

Let $x = 0$

$0 + 1 = -A + 0$

$A = -1$

Let $x = \frac{1}{3}$

$\frac{1}{3} + 1 = 0 + \frac{1}{3}B$

$\frac{4}{3} = \frac{1}{3}B$

$4 = B$

$\therefore \frac{x+1}{x(3x-1)} = -\frac{1}{x} + \frac{4}{(3x-1)}$

2) a) $\frac{2x^2+3}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$

$= \frac{A(x-1)(x+2) + B(x+2) + C(x-1)^2}{(x-1)^2(x+2)}$

Let $x = 1$

$2 + 3 = 0 + 3B + 0$

$\frac{5}{3} = B$

Let $x = -2$

$8 + 3 = 0 + 0 + 9C$

$\frac{11}{9} = C$

Let $x = 0$

$0 + 3 = A(-1)(2) + B(2) + C(-1)^2$

$3 = -2A + 2B + C$

$3 = -2A + 2(5/3) + 11/9$

$3 = -2A + 10/3 + 11/9$

$2A = 11/9 + 30/9 - \frac{27}{9}$

$2A = 14/9$

$A = 7/9$

$\therefore \frac{2x^2+3}{(x-1)^2(x+2)} = \frac{7}{9(x-1)} + \frac{5}{3(x-1)^2} + \frac{11}{9(x+2)}$

b) $\frac{3x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)}$

$= \frac{Ax(x+1) + B(x+1) + Cx^2}{x^2(x+1)}$

Let $x = 0$

$0 - 1 = 0 + B + 0$

$-1 = B$

Let $x = -1$

$-3 - 1 = 0 + 0 + C$

$-4 = C$

Let $x = 2$

$6 - 1 = A(2)(3) + B(3) + C(4)$

$5 = 6A + 3B + 4C$

$5 = 6A - 3 - 16$

$24 = 6A$

$4 = A$

$\therefore \frac{3x-1}{x^2(x+1)} = \frac{4}{x} - \frac{1}{x^2} - \frac{4}{(x+1)}$

(3) a) $\dfrac{8x^2+x-5}{(2x-1)^2(x+2)} = \dfrac{A}{(2x-1)} + \dfrac{B}{(2x-1)^2} + \dfrac{C}{(x+2)}$

$= \dfrac{A(2x-1)(x+2) + B(x+2) + C(2x-1)^2}{(2x-1)^2(x+2)}$

Let $x = -2$

$32 - 2 - 5 = 0 + 0 + 25C$

$25 = 25C$

$1 = C$

Let $x = 1/2$

$2 + \frac{1}{2} - 5 = 0 + B\left(\frac{5}{2}\right) + 0$

$\frac{-5}{2} = \frac{5}{2}B$

$-1 = B$

Let $x = 0$

$0 + 0 - 5 = A(-1)(2) + B(2) + C(1)$

$-5 = -2A + 2B + C$

$-5 = -2A - 2 + 1$

$2A = 4$

$A = 2$

$\therefore \dfrac{8x^2+x-5}{(2x-1)^2(x+2)} = \dfrac{2}{(2x-1)} - \dfrac{1}{(2x-1)^2} + \dfrac{1}{(x+2)}$

b) $\therefore \int \dfrac{8x^2+x-5}{(2x-1)^2(x+2)}\,dx = \int \dfrac{2}{2x-1}\,dx - \int (2x-1)^{-2}\,dx + \int \dfrac{1}{x+2}\,dx$

$= \ln|2x-1| - \dfrac{(2x-1)^{-1}}{(-1)\times 2} + \ln|x+2| + C$

$= \ln|2x-1| + \dfrac{1}{2(2x-1)} + \ln|x+2| + C$

(4) a) $f(x) = \dfrac{2x^2+4}{(x-2)^2(x+4)} = \dfrac{A}{(x-2)} + \dfrac{B}{(x-2)^2} + \dfrac{C}{(x+4)}$

$= \dfrac{A(x-2)(x+4) + B(x+4) + C(x-2)^2}{(x-2)^2(x+4)}$

Let $x = 2$

$8 + 4 = 0 + 6B + 0$

$12 = 6B$

$2 = B$

Let $x = -4$

$32 + 4 = 0 + 0 + C(36)$

$36 = 36C$

$1 = C$

Let $x = 0$

$0 + 4 = A(-2)(4) + B(4) + C(4)$

$4 = -8A + 4(2) + 4(1)$

$8A = 8$

$A = 1$

$\therefore \dfrac{2x^2+4}{(x-2)^2(x+4)} = \dfrac{1}{(x-2)} + \dfrac{2}{(x-2)^2} + \dfrac{1}{(x+4)}$

b) $f(x) = (x-2)^{-1} + 2(x-2)^{-2} + (x+4)^{-1}$

$f'(x) = -(x-2)^{-2} + 2(-2)(x-2)^{-3} - (x+4)^{-2}$

$= -\dfrac{1}{(x-2)^2} - \dfrac{4}{(x-2)^3} - \dfrac{1}{(x+4)^2}$

$\therefore f'(0) = -\dfrac{1}{4} - \dfrac{4}{(-8)} - \dfrac{1}{16}$

$= -\dfrac{4}{16} + \dfrac{8}{16} - \dfrac{1}{16}$

$= \dfrac{3}{16}$