Trigonometry 2 : Answers

1)
$$13\tan^2\theta = 5\sec^2\theta + 6\tan\theta$$
$$13\tan^2\theta = 5(1+\tan^2\theta) + 6\tan\theta$$
$$13\tan^2\theta = 5 + 5\tan^2\theta + 6\tan\theta$$
$$8\tan^2\theta - 6\tan\theta - 5 = 0$$
$$(4\tan\theta - 5)(2\tan\theta + 1) = 0$$

either $4\tan\theta - 5 = 0$ or $2\tan\theta + 1 = 0$

$\tan\theta = 4/5$ $\qquad$ $\tan\theta = -1/2$

$\alpha = 38.7^\circ$ $\qquad$ $\alpha = 26.6^\circ$

tan +ve 1st + 3rd $\qquad$ tan −ve 2nd + 4th

$\theta = 38.7^\circ, 218.7^\circ$ $\qquad$ $\theta = 153.4^\circ, 333.4^\circ$

$$\theta = 38.7^\circ, 153.4^\circ, 218.7^\circ, 333.4^\circ$$

2)
$$3\operatorname{cosec}^2\theta = 11 - 2\cot\theta$$
$$3(1+\cot^2\theta) = 11 - 2\cot\theta$$
$$3 + 3\cot^2\theta = 11 - 2\cot\theta$$
$$3\cot^2\theta + 2\cot\theta - 8 = 0$$
$$(3\cot\theta - 4)(\cot\theta + 2) = 0$$

either $3\cot\theta - 4 = 0$ or $\cot\theta + 2 = 0$

$\cot\theta = 4/3$ $\qquad$ $\cot\theta = -2$

$\tan\theta = \frac{3}{4}$ $\qquad$ $\tan\theta = -\frac{1}{2}$

$\alpha = 36.9^\circ$ $\qquad$ $\alpha = 26.6^\circ$

tan +ve 1st + 3rd $\qquad$ tan −ve 2nd + 4th

$\theta = 36.9^\circ, 216.9^\circ$ $\qquad$ $\theta = 153.4^\circ, 333.4^\circ$

$$\therefore\ \theta = 36.9^\circ, 153.4^\circ, 216.9^\circ, 333.4^\circ$$

3)
$$4\operatorname{cosec}^2\theta = 9 - 8\cot\theta$$
$$4(1+\cot^2\theta) = 9 - 8\cot\theta$$
$$4 + 4\cot^2\theta = 9 - 8\cot\theta$$
$$4\cot^2\theta + 8\cot\theta - 5 = 0$$
$$(2\cot\theta + 5)(2\cot\theta - 1) = 0$$

either $\cot\theta = -5/2$ or $\cot\theta = 1/2$

$\tan\theta = -2/5$ $\qquad$ $\tan\theta = 2$

$\alpha = 21.8^\circ$ $\qquad$ $\alpha = 63.4^\circ$

tan −ve 2nd and 4th $\qquad$ tan +ve 1st + 3rd

$\theta = 158.2^\circ, 338.2^\circ$ $\qquad$ $\theta = 63.4^\circ, 243.4^\circ$

$$\therefore\ \theta = 63.4^\circ, 158.2^\circ, 243.4^\circ, 338.2^\circ$$

4) $\cot^2\theta = 7 - 2\operatorname{Cosec}\theta$ $\qquad 1 + \cot^2\theta = \operatorname{Cosec}^2\theta$

$\cot^2\theta = \operatorname{Cosec}^2\theta - 1$

$\operatorname{Cosec}^2\theta - 1 = 7 - 2\operatorname{Cosec}\theta$

$\operatorname{Cosec}^2\theta + 2\operatorname{Cosec}\theta - 8 = 0$

$(\operatorname{Cosec}\theta + 4)(\operatorname{Cosec}\theta - 2) = 0$

either $\operatorname{Cosec}\theta = -4$ or $\operatorname{Cosec}\theta = 2$

$\sin\theta = -\frac{1}{4}$ $\qquad \sin\theta = \frac{1}{2}$

$\alpha = 14.5^\circ$ $\qquad \alpha = 30^\circ$

Sin -ve 3rd + 4th $\qquad$ Sin +ve 1st + 2nd

$\theta = 194.5^\circ, 345.5^\circ$ $\qquad \theta = 30^\circ, 150^\circ$

$\therefore \theta = 30^\circ, 150^\circ, 194.5^\circ, 345.5^\circ$

5) $3\tan^2\theta = 7 + \sec\theta$ $\qquad 1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$3(\sec^2\theta - 1) = 7 + \sec\theta$

$3\sec^2\theta - 3 = 7 + \sec\theta$

$3\sec^2\theta - \sec\theta - 10 = 0$

$(3\sec\theta + 5)(\sec\theta - 2) = 0$

either $\qquad$ or

$\sec\theta = -5/3$ $\qquad \sec\theta = 2$

$\cos\theta = -\frac{3}{5}$ $\qquad \cos\theta = \frac{1}{2}$

$\alpha = 53.1^\circ$ $\qquad \alpha = 60^\circ$

Cos -ve 2nd + 3rd $\qquad$ Cos +ve 1st + 4th

$\theta = 126.9^\circ, 233.1^\circ$ $\qquad \theta = 60^\circ, 300^\circ$

$\therefore \theta = 60^\circ, 126.9^\circ, 233.1^\circ, 300^\circ$

6) $2\tan^2\theta = \sec\theta + 8$ $\qquad 1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$2(\sec^2\theta - 1) = \sec\theta + 8$

$2\sec^2\theta - 2 = \sec\theta + 8$

$2\sec^2\theta - \sec\theta - 10 = 0$

$(2\sec\theta - 5)(\sec\theta + 2) = 0$

either $\qquad$ or

$\sec\theta = 5/2$ $\qquad \sec\theta = -2$

$\cos\theta = 2/5$ $\qquad \cos\theta = -\frac{1}{2}$

$\alpha = 66.4^\circ$ $\qquad \alpha = 60^\circ$

Cos +ve 1st + 4th $\qquad$ Cos -ve 2nd + 3rd

$\theta = 66.4^\circ, 293.6^\circ$ $\qquad \theta = 120^\circ, 240^\circ$

$\therefore \theta = 66.4^\circ, 120^\circ, 240^\circ, 293.6^\circ$

7) $7\operatorname{Cosec}^2\theta - 4\operatorname{Cot}^2\theta = 16 + 5\operatorname{Cosec}\theta$ $\qquad 1 + \operatorname{Cot}^2\theta = \operatorname{Cosec}^2\theta$

$7\operatorname{Cosec}^2\theta - 4(\operatorname{Cosec}^2\theta - 1) = 16 + 5\operatorname{Cosec}\theta$ $\qquad \operatorname{Cot}^2\theta = \operatorname{Cosec}^2\theta - 1$

$7\operatorname{Cosec}^2\theta - 4\operatorname{Cosec}^2\theta + 4 = 16 + 5\operatorname{Cosec}\theta$

$3\operatorname{Cosec}^2\theta - 5\operatorname{Cosec}\theta - 12 = 0$

$(3\operatorname{Cosec}\theta + 4)(\operatorname{Cosec}\theta - 3) = 0$

either $\operatorname{Cosec}\theta = -4/3$ or $\operatorname{Cosec}\theta = 3$

$\sin\theta = -\frac{3}{4}$ $\qquad \sin\theta = \frac{1}{3}$

$\alpha = 48.6^\circ$ $\qquad \alpha = 19.5^\circ$

Sin -ve 3rd + 4th $\qquad$ Sin +ve 1st + 2nd

$\theta = 228.6^\circ, 311.4^\circ$ $\qquad \theta = 19.5^\circ, 160.5^\circ$

$\therefore \theta = 19.5^\circ, 160.5^\circ, 228.6^\circ, 311.4^\circ$

8) $3\operatorname{Cosec}\theta(\operatorname{Cosec}\theta - 1) = 5\operatorname{Cot}^2\theta - 9$ $\qquad 1 + \operatorname{Cot}^2\theta = \operatorname{Cosec}^2\theta$

$3\operatorname{Cosec}^2\theta - 3\operatorname{Cosec}\theta = 5(\operatorname{Cosec}^2\theta - 1) - 9$ $\qquad \operatorname{Cot}^2\theta = \operatorname{Cosec}^2\theta - 1$

$3\operatorname{Cosec}^2\theta - 3\operatorname{Cosec}\theta = 5\operatorname{Cosec}^2\theta - 5 - 9$

$0 = 2\operatorname{Cosec}^2\theta + 3\operatorname{Cosec}\theta - 14$

$0 = (2\operatorname{Cosec}\theta + 7)(\operatorname{Cosec}\theta - 2)$

either $\operatorname{Cosec}\theta = -7/2$ or $\operatorname{Cosec}\theta = 2$

$\sin\theta = -2/7$ $\qquad \sin\theta = 1/2$

$\alpha = 16.6^\circ$ $\qquad \alpha = 30^\circ$

Sin -ve 3rd + 4th $\qquad$ Sin +ve 1st + 2nd

$\theta = 196.6^\circ, 343.4^\circ$ $\qquad \theta = 30^\circ, 150^\circ$

$\therefore \theta = 30^\circ, 150^\circ, 196.6^\circ, 343.4^\circ$

9) $3\sec^2\theta = 7 - 11\tan\theta$

$3(1 + \tan^2\theta) = 7 - 11\tan\theta$

$3 + 3\tan^2\theta = 7 - 11\tan\theta$

$3\tan^2\theta + 11\tan\theta - 4 = 0$

$(3\tan\theta - 1)(\tan\theta + 4) = 0$

either $\tan\theta = 1/3$ or $\tan\theta = -4$

$\alpha = 18.4^\circ$ $\qquad \alpha = 76.0^\circ$

tan +ve 1st + 3rd $\qquad$ tan -ve 2nd + 4th

$\theta = 18.4^\circ, 198.4^\circ$ $\qquad \theta = 104.0^\circ, 286.0^\circ$

$\therefore \theta = 18.4^\circ, 104.0^\circ, 198.4^\circ, 286.0^\circ$

10) $6\tan^2\theta - 6 = 4\sec^2\theta + 5\sec\theta$ $\qquad 1 + \tan^2\theta = \sec^2\theta$

$6(\sec^2\theta - 1) - 6 = 4\sec^2\theta + 5\sec\theta$ $\qquad \tan^2\theta = \sec^2\theta - 1$

$6\sec^2\theta - 6 - 6 = 4\sec^2\theta + 5\sec\theta$

$2\sec^2\theta - 5\sec\theta - 12 = 0$

$(2\sec\theta + 3)(\sec\theta - 4) = 0$

either $\sec\theta = -3/2$ or $\sec\theta = 4$

$\cos\theta = -2/3$ $\qquad \cos\theta = 1/4$

$\alpha = 48.2^\circ$ $\qquad \alpha = 75.5^\circ$

Cos -ve 2nd + 3rd $\qquad$ Cos +ve 1st + 4th

$\theta = 131.8^\circ, 228.2^\circ$ $\qquad \theta = 75.5^\circ, 284.5^\circ$

$\therefore \theta = 75.5^\circ, 131.8^\circ, 228.2^\circ, 284.5^\circ$

11) $\operatorname{cosec}^2 x + \cot^2 x = 3$

$1 + \cot^2 x + \cot^2 x = 3$

$2\cot^2 x - 2 = 0$

$\div 2$ $\quad \cot^2 x - 1 = 0$

$(\cot x - 1)(\cot x + 1) = 0$

either $\cot x = 1$ or $\cot x = -1$

$\tan x = 1$ $\qquad \tan x = -1$

$\alpha = 45^\circ$ $\qquad \alpha = 45^\circ$

tan +ve 1st + 3rd $\qquad$ tan -ve 2nd + 4th

$x = 45^\circ, 225^\circ$ $\qquad x = 135^\circ, 315^\circ$

$\therefore x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$