Trigonometry 4 : Double Angle Identity Eqns : Answers

1) $6\cos 2x + \sin x - 6 = 0$

$6(1 - 2\sin^2 x) + \sin x - 6 = 0$

$6 - 12\sin^2 x + \sin x - 6 = 0$

$0 = 12\sin^2 x - \sin x$

$0 = \sin x (12\sin x - 1)$

either

$\sin x = 0$ or $\sin x = 1/2$

$x = 0°, 180°, 360°$ $\quad \alpha = 4{\cdot}8°$

Sin +ve 1st + 2nd

$x = 4{\cdot}8°, 184{\cdot}8°$

$\therefore x = 0°, 4{\cdot}8°, 180°, 184{\cdot}8°, 360°$

2) $2 + 3\cos 2\theta = \cos\theta$

$2 + 3(2\cos^2\theta - 1) = \cos\theta$

$2 + 6\cos^2\theta - 3 = \cos\theta$

$6\cos^2\theta - \cos\theta - 1 = 0$

$(3\cos\theta + 1)(2\cos\theta - 1) = 0$

either $\cos\theta = -\frac{1}{3}$ or $\cos\theta = 1/2$

$\alpha = 70{\cdot}6°$ $\quad \alpha = 60°$

Cos -ve 2nd + 3rd Cos +ve 1st + 4th

$\theta = 109{\cdot}4°, 250{\cdot}6°$ $\quad \theta = 60°, 300°$

$\therefore \theta = 60°, 109{\cdot}4°, 250{\cdot}6°, 300°$

3) $\tan 2x = 3\cot x$

$\frac{2\tan x}{1 - \tan^2 x} = \frac{3}{\tan x}$

$2\tan^2 x = 3(1 - \tan^2 x)$

$2\tan^2 x = 3 - 3\tan^2 x$

$5\tan^2 x = 3$

$\tan^2 x = \frac{3}{5}$

either $\tan x = +\frac{\sqrt{3}}{\sqrt{5}}$ or $\tan x = -\frac{\sqrt{3}}{\sqrt{5}}$

$\alpha = 37{\cdot}8°$ $\quad \alpha = 37{\cdot}8°$

tan +ve 1st + 3rd tan -ve 2nd + 4th

$x = 37{\cdot}8°, 217{\cdot}8°$ $\quad x = 142{\cdot}2°, 322{\cdot}2°$

$\therefore x = 37{\cdot}8°, 142{\cdot}2°$ for given range

4) $5\cos^2\theta + 7\sin 2\theta = 3\sin^2\theta$

$5\cos^2\theta + 7(2\sin\theta\cos\theta) = 3\sin^2\theta$

$\div \cos^2\theta$

$5 + \frac{14\sin\theta}{\cos\theta} = 3\tan^2\theta$

$0 = 3\tan^2\theta - 14\tan\theta - 5$

$\therefore$

$a = 3 \quad b = -14 \quad c = -5$

$0 = (3\tan\theta + 1)(\tan\theta - 5)$

either $\tan\theta = -\frac{1}{3}$ or $\tan\theta = 5$

$\alpha = 18.4°$ $\quad$ $\alpha = 78.7°$

tan -ve 2nd + ~~4th~~ $\quad$ tan +ve 1st + ~~3rd~~

$\theta = 161.6°$ $\quad$ $\theta = 78.7°$

$\therefore \theta = 78.7°, 161.6°$

5) $\sin 2x - 2\cos^2 x = 0$

$2\sin x\cos x - 2\cos^2 x = 0$

$\div 2$ $\quad \sin x\cos x - \cos^2 x = 0$

$\cos x(\sin x - \cos x) = 0$

either $\quad$ or $\quad \sin x = \cos x$

$\cos x = 0$ $\quad$ $\tan x = 1$

$x = 90°, 270°$ $\quad$ $\alpha = 45°$

tan +ve 1st + 3rd

$x = 45°, 225°$

$\therefore x = 45°, 90°, 225°, 270°$

6) $5 - 13\sin\gamma = 2\cos 2\gamma$

$5 - 13\sin\gamma = 2(1 - 2\sin^2\gamma)$

$5 - 13\sin\gamma = 2 - 4\sin^2\gamma$

$4\sin^2\gamma - 13\sin\gamma + 3 = 0$

$(\sin\gamma - 3)(4\sin\gamma - 1) = 0$

either $\sin\gamma = 3$ $\quad$ or $\quad \sin\gamma = 1/4$

IMPOSSIBLE (NO SOLUTIONS) $\quad$ $\alpha = 14.5°$

Sin +ve 1st + 2nd

$\gamma = 14.5°, 165.5°$