Cubics 2 : Answers

1) a) $x^3 + 2x^2 - x - 2$

$f(-2) = -8 + 2(4) - (-2) - 2 = -8 + 8 + 2 - 2$
$= 0$

$\therefore$ $x+2$ is a factor

b) $3x^3 - 2x^2 + x + 2$

$f(2) = 3(8) - 2(4) + 2 + 2$
$= 24 - 8 + 2 + 2 \neq 0$

$\therefore$ $x-2$ is NOT a factor

c) $x^3 + x^2 + x - 2$

$f(1) = 1 + 1 + 1 - 2 \neq 0$

$\therefore$ $x-1$ is NOT a factor

d) $x^3 - 5x^2 - 2x + 24$

$f(3) = 27 - 5(9) - 2(3) + 24$
$= 27 - 45 - 6 + 24$
$= 0$ $\therefore$ $x-3$ is a factor

e) $6x^3 + 5x^2 - 3x - 2$

$f(-2) = 6(-8) + 5(4) - 3(-2) - 2$
$= -48 + 20 + 6 - 2 \neq 0$

$\therefore$ $x+2$ is NOT a factor

f) $x^3 - 3$

$f(-2) = -8 - 3 \neq 0$

$\therefore$ $x+2$ is NOT a factor

g) $9x^3 + 18x^2 - 4x - 8$

$f(-2) = 9(-8) + 18(4) - 4(-2) - 8$
$= 0$ $\therefore$ $x+2$ is a factor

2) a) $4x^3 - 21x - 10$

$f(1) = 4 - 21 - 10 \neq 0$
$f(-1) = -4 + 21 - 10 \neq 0$
$f(2) = 32 - 42 - 10 \neq 0$
$f(-2) = -32 + 42 - 10 = 0$

$\therefore$ $x+2$ is a factor

Use coefficients method

$$4x^3 - 21x - 10 = (x+2)(ax^2 + bx + c)$$

Compare x^3
$4 = a$

Compare consts
$-10 = 2c$
$-5 = c$

Compare x^2
$0 = b + 2a$
$0 = b + 8$
$-8 = b$

$$\therefore 4x^3 - 21x - 10 = (x+2)(4x^2 - 8x - 5)$$
$$= (x+2)(2x+1)(2x-5)$$

b) $6x^3 - 7x^2 - 14x + 8$
$f(1) = 6 - 7 - 14 + 8 \neq 0$
$f(-1) = -6 - 7 + 14 + 8 \neq 0$
$f(2) = 48 - 28 - 28 + 8 = 0$
$\therefore$ $x-2$ is a factor

Coefficeents method

$$6x^3 - 7x^2 - 14x + 8 = (x-2)(ax^2 + bx + c)$$

Compare x^3
$6 = a$

Compare consts
$8 = -2c$
$-4 = c$

Compare x^2
$-7 = -2a + b$
$-7 = -12 + b$
$5 = b$

$$\therefore 6x^3 - 7x^2 - 14x + 8 = (x-2)(6x^2 + 5x - 4)$$
$$= (x-2)(3x+4)(2x-1)$$

c) $x^3 + 2x^2 - x - 2$

$f(1) = 1 + 2 - 1 - 2 \quad = 0$

$\therefore \quad x - 1$ is a factor

Use Long Division Method

$$
\begin{array}{r|l}
 & x^2 + 3x + 2 \\
\hline
x - 1 & x^3 + 2x^2 - x - 2 \\
 & -\,x^3 \pm x^2 \\
\hline
 & \quad 3x^2 - x \\
 & -\,3x^2 \overset{+}{-} 3x \\
\hline
 & \qquad 2x - 2 \\
 & -\,2x \pm 2 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$\therefore \quad x^3 + 2x^2 - x - 2 \quad = (x-1)(x^2 + 3x + 2)$

$= (x-1)(x+2)(x+1)$

d) $x^3 + 3x^2 - x - 3$

$f(1) = 1 + 3 - 1 - 3$

$\therefore \quad x - 1$ is a factor

Long Division Method

$$
\begin{array}{r|l}
 & x^2 + 4x + 3 \\
\hline
x - 1 & x^3 + 3x^2 - x - 3 \\
 & -\,x^3 \pm x^2 \\
\hline
 & \quad 4x^2 - x \\
 & -\,4x^2 \overset{+}{-} 4x \\
\hline
 & \qquad 3x - 3 \\
 & -\,3x \overset{+}{-} 3 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$\therefore \quad x^3 + 3x^2 - x - 3 \quad = (x-1)(x^2 + 4x + 3)$

$= (x-1)(x+3)(x+1)$

e) $x^3 - 3x^2 - x + 3$

$f(1) = 1 - 3 - 1 + 3 = 0$

$\therefore$ $x - 1$ is a factor

Use coefficients method

$$x^3 - 3x^2 - x + 3 = (x-1)(ax^2 + bx + c)$$

<u>Compare x^3</u>
$1 = a$

<u>Compare consts</u>
$3 = -c$
$-3 = c$

<u>Compare x^2</u>
$-3 = -a + b$
$-3 = -1 + b$
$-2 = b$

$$\therefore x^3 - 3x^2 - x + 3 = (x-1)(x^2 - 2x - 3)$$
$$= (x-1)(x+1)(x-3)$$

3) Solve

a) $4x^3 - 21x - 10 = 0$

from 2a) $(x+2)(2x+1)(2x-5) = 0$

either $x + 2 = 0$, $x = -2$ or $2x + 1 = 0$, $x = -1/2$ or $2x - 5 = 0$, $x = 5/2$

b) $6x^3 - 7x^2 - 14x + 8 = 0$

from 2b) $(x-2)(3x+4)(2x-1) = 0$

either $x - 2 = 0$, $x = 2$ or $3x + 4 = 0$, $x = -4/3$ or $2x - 1 = 0$, $x = 1/2$

c) $x^3 + 2x^2 - x - 2 = 0$

from 2c) $(x-1)(x+2)(x+1) = 0$

either $x - 1 = 0$, $x = 1$ or $x + 2 = 0$, $x = -2$ or $x + 1 = 0$, $x = -1$

d) $x^3 + 3x^2 - x - 3 = 0$

from 2d) $(x-1)(x+3)(x+1) = 0$

either $x - 1 = 0$, $x = 1$ or $x + 3 = 0$, $x = -3$ or $x + 1 = 0$, $x = -1$

e) $x^3 - 3x^2 - x + 3 = 0$

from 2e) $(x-1)(x+1)(x-3) = 0$

either $x - 1 = 0$, $x = 1$ or $x + 1 = 0$, $x = -1$ or $x - 3 = 0$, $x = 3$.