Dynamics with Calculus 1 : Answers

1) $V = 8t^3 - 6t + 2$

a) $\frac{dx}{dt} = 8t^3 - 6t + 2$

$x = 2t^4 - 3t^2 + 2t + c$

$t = 0 \quad x = 0$

$0 = 0 - 0 + 0 + c$

$0 = c$

$\therefore \quad x = 2t^4 - 3t^2 + 2t$

$\underline{t = 1}$

$x = 2 - 3 + 2$

$x = +1 \text{ m} \quad \therefore$ 1m from origin.

b) $a = \frac{dv}{dt} = 24t^2 - 6$

$t = 2$

$a = 24(4) - 6$

$a = 96 - 6$

$a = 90 \text{ m/s}^2$

2) $V = 3t^2 - 24t + 21$

a) $a = \frac{dv}{dt} = 6t - 24$

b) $v = \frac{dx}{dt} = 3t^2 - 24t + 21$

$x = t^3 - 12t^2 + 21t + c$

$x = 0 \quad t = 0$

$0 = 0 - 0 + 0 + c$

$0 = c$

$\therefore \quad x = t^3 - 12t^2 + 21t$

c) At rest $v = 0 \quad \therefore \quad 0 = 3t^2 - 24t + 21$

$\div 3 \quad 0 = t^2 - 8t + 7$

$0 = (t - 1)(t - 7)$

$\therefore \quad t = 1 \text{ sec} \quad \text{or} \quad t = 7 \text{ secs}$

$\therefore$ At rest after 1 sec and 7 secs.

$t = 1$

$x = 1 - 12 + 21$

$x = +10\ m$

$t = 7$

$x = 7^3 - 12(7^2) + 21(7)$

$x = 343 - 588 + 147$

$x = -98\ m$

$\therefore$ Distance between them

is 108 m

3) $v = 12t - 3t^2$

a) $\frac{dx}{dt} = 12t - 3t^2$

$x = 6t^2 - t^3 + c$

$t = 1 \quad x = 0$

$0 = 6 - 1 + c$

$-5 = c$

$\therefore \quad x = 6t^2 - t^3 - 5$

b) $a = \frac{dv}{dt} = 12 - 6t$

4) $v = 2t(t-6)$

$v = 2t^2 - 12t$

a) $2t^2 - 12t < 0$

$t^2 - 6t < 0$

$t(t-6) < 0$

either $(+) \times (-) \Rightarrow (-)$ OR $(-) \times (+) \Rightarrow (-)$

$t > 0$ and $t - 6 < 0$

$t > 0 \quad t < 6$

$0 < t < 6$

$t < 0$ and $t - 6 > 0$

$t < 0 \quad t > 6$

IMPOSSIBLE

b) Split into 2 regions 0 to 6 secs (travels ←)

6 to 9 secs (travels →)

b) continued

$$\frac{dx}{dt} = 2t^2 - 12t$$

$$x = \frac{2t^3}{3} - 6t^2 + c$$

$t = 0 \quad x = 0$ (away from start position ... not away from origin)

$$0 = 0 - 0 + c$$
$$0 = c$$

$$\therefore \quad x = \frac{2t^3}{3} - 6t^2$$

$t = 9 \qquad x = \frac{2}{3} \times 9^3 - 6 \times 9^2$

$$x = 486 - 486$$
$$x = 0$$

$t = 6 \qquad x = \frac{2}{3} \times 6^3 - 6 \times 6^2$

$$x = 144 - 216$$
$$x = -72 \text{ m}$$

so journey 0 to 6 secs sees particle moving 72m to left away from start

Then from 6 to 9 secs it travels to right, doubling back on itself.
By $t = 9$ it has returned to start position.

∴ Total distance travelled in the interval $0 \leq t \leq 9$

is 72 (0 to 6 secs) + 72 (6 to 9 secs)

72 ↑ left, 72 ↑ right

$= 144$ m

5) $V = 4t^3 - 6t + 7 = \frac{dx}{dt}$ $\quad m = 0.8\,kg$

a) $x = t^4 - 3t^2 + 7t + C$

$t = 0 \quad x = 5$

$5 = 0 - 0 + 0 + C$

$5 = C$

$\therefore x = t^4 - 3t^2 + 7t + 5$

$\underline{t=2}$

$x = 16 - 12 + 14 + 5$

$x = 23\ m$

b) $a = \frac{dv}{dt} = 12t^2 - 6$

$t = 3$

$a = 12(9) - 6$

$a = 102\ m/s^2$

$\therefore F = ma$ when $t = 3$

$F = 0.8(102)$

$F = 81.6\ N$

6) $V = 5t^4 + 3t^2$

a) $a = \frac{dv}{dt} = 20t^3 + 6t$

$\underline{t=1}$

$a = 20 + 6$

$a = 26\ m/s^2$

b) $V = \frac{dx}{dt} = 5t^4 + 3t^2$

$x = t^5 + t^3 + C$

$t = 0 \quad x = 1$

$1 = 0 + 0 + C$

$1 = C$

$\therefore x = t^5 + t^3 + 1$

When $x = 2$ $\quad x = 32 + 8 + 1$

$x = 41\ m$