Dynamics with Calculus 2 : Answers

1) $v = \frac{1}{60}t(60-t)$

$v = t - \frac{t^2}{60}$

a) stationary when $v = 0$

$0 = t - \frac{t^2}{60}$

$0 = \frac{1}{60}t(60-t)$

either $t = 0$ or $t = 60$

b) $x = \frac{t^2}{2} - \frac{t^3}{180} + C$

$t = 0 \quad x = 0$

$0 = 0 - 0 + c$

$0 = c$

$\therefore x = \frac{t^2}{2} - \frac{t^3}{180}$

$t = 60$

$x = \frac{60^2}{2} - \frac{60^3}{180}$

$x = 1800 - 1200$

$x = 600$

$\therefore$ Has travelled 600m from $t=0$ to $t=60$

c) (i) $a = \frac{dv}{dt} = 1 - \frac{t}{30}$

(ii) Greatest speed when acc = 0

$\therefore 1 - \frac{t}{30} = 0$

$1 = \frac{t}{30}$

30 secs = t gives greatest speed.

$t = 30 \quad v = 30 - \frac{30^2}{60}$

$v = 30 - 15 = 15\,m/s$ is greatest speed.

2. $v = \frac{1}{20}(3t^2 + 4t + 5)$

a) $a = \frac{dv}{dt} = \frac{1}{20}(6t + 4)$

Now find t when $\frac{1}{20}(6t+4) = 5$

$6t + 4 = 100$

$6t = 96$

$t = 16$ secs.

b) $v = \frac{dx}{dt} = \frac{1}{20}(3t^2+4t+5)$

$x = \frac{1}{20}(t^3 + 2t^2 + 5t) + C$

Now when $t = 0$ $x = 0$

↖ displacement from start position wherever the start is, is irrelevant for this value.

$0 = 0 + C$

$0 = C$

$x = \frac{1}{20}(t^3 + 2t^2 + 5t)$

Because of +ve signs in original vel expression, all velocities for 0 to 20 secs are +ve ie →

∴ Displacement from start position will simply be the x value when $t = 20$

$t = 20$ $x = \frac{1}{20}(20^3 + 2(20^2) + 5(20))$

Expand

$x = 20^2 + 2(20) + 5$

$x = 400 + 40 + 5$

$x = 445$ m

3. $V = \frac{3t^2 - 4}{3t}$ $\quad t > 0$

$V = \frac{t^2}{3t} - \frac{4}{3t}$

$V = \frac{t}{3} - \frac{4t^{-1}}{3}$

a) find t when $V = 0$

$\frac{t^2}{3t} - \frac{4}{3t} = 0$

$\frac{t}{3} - \frac{4}{3t} = 0$

$\times 3t$

$t^2 - 4 = 0$

$(t-2)(t+2) = 0$

either $t = 2$ or $t = -2$

$\therefore$ At rest after 2 seconds

b) For $t = 2$ to $t = 5$

all velocity values ≥ 0

(TRY SUB IN $t=2$ $t=3$ $t=4$ $t=5$ and you can see all vel values are ≥ 0)

So motion is $\longrightarrow$ for all times $t=2$ to $t=5$

$\therefore$ Find x value $t=2$ x value $t=5$ and find the difference.

$\frac{dx}{dt} = \frac{t}{3} - \frac{4}{3}t^{-1}$

$x = \frac{t^2}{6} - \frac{4}{3}\ln t + c$

$t=0$ $x=0$

$x = \frac{t^2}{6} - \frac{4}{3}\ln t$

↑ displacement from start

$t = 2$

$x = \frac{2^2}{6} - \frac{4}{3}\ln 2$

$x = \frac{2}{3} - 0.98083$

$x = -0.31416$

$t = 5$

$x = \frac{5^2}{6} - \frac{4}{3}\ln 5$

$x = +2.26954$

from 0 to 2 secs it moved left from the start position 0.31416 m

from 2 to 5 secs it moves right to reach 2.26954 m right of the start.

∴ Distance travelled from $t=2$ to $t=5$

$= 0.31416 + 2.26954$

$= 2.5837$

≈ 2.584 m covered.

4) $V = 12t^2 - 7kt + 1$

$x = 4t^3 - \frac{7kt^2}{2} + t + c$

a)

$t = 0 \quad x = 3$

$3 = 0 - 0 + 0 + c$

$3 = c$

$x = 4t^3 - \frac{7k}{2}t^2 + t + 3$

$t = 2 \quad x = 16$

$16 = 32 - 14k + 2 + 3$

$16 = 37 - 14k$

$14k = 21$

$K = \frac{3}{2}$

$v = 12t^2 - \frac{21}{2}t + 1$

b) $a = \frac{dv}{dt} = 24t - \frac{21}{2}$

$t = 5 \quad a = 24(5) - \frac{21}{2}$

$a = 120 - \frac{21}{2}$

$a = \frac{219}{2} \ m/s^2$

$\therefore$ Force

$F = ma$

$F = 4 \times \frac{219}{2}$

$F = 438 \ N$