Dynamics With Calculus 3 : Answers

1) $V = \frac{1}{20}(80 + 16t - t^2)$

a) $a = \frac{dv}{dt} = \frac{1}{20}(16 - 2t)$

b) Max velocity is when $acc = 0$

OR you could say use stationary point work.

$$\frac{dV}{dt} = \frac{1}{20}(16 - 2t) = 0 \text{ at S.P's}$$

$$16 - 2t = 0$$
$$16 = 2t$$
$$8 \text{ secs} = t$$

$$\therefore \text{Max Vel} = \frac{1}{20}(80 + 16(8) - 8^2)$$
$$= \frac{1}{20}(80 + 128 - 64)$$
$$= \frac{1}{20}(144)$$
$$= 7.2 \text{ m/s}$$

$$\frac{d^2V}{dt^2} = \frac{1}{20}(-2)$$
$$= -ve \quad \therefore \text{ Maximum value}$$

c)
$$\frac{dx}{dt} = V = \frac{1}{20}(80 + 16t - t^2)$$
$$x = \frac{1}{20}\left(80t + 8t^2 - \frac{t^3}{3}\right) + c$$

$x = 0$ when $t = 0$ $\quad \therefore \quad 0 = 0 + c$

$$0 = c$$

$$\therefore \quad x = \frac{1}{20}\left(80t + 8t^2 - \frac{t^3}{3}\right)$$

when $t = 20$

$$x = \frac{1}{20}\left(80(20) - 8(20^2) - \frac{20^3}{3}\right)$$

$$x = \frac{1}{20}\left(1600 - 3200 - \frac{8000}{3}\right)$$

$$x = -\frac{640}{3} \text{ m}$$

ie 213.3 m left of A.

2) $F = 30t^{-2} - 30$ N

a) $\xrightarrow{acc}$

$$120\text{N} \leftarrow \bigcirc \rightarrow \frac{30}{t^2} - 30 \text{ N}$$

$$RF = ma$$

$$30t^{-2} - 30 - 120 = 5\frac{dv}{dt}$$

$$30t^{-2} - 150 = 5\frac{dv}{dt}$$

$$6t^{-2} - 30 = \frac{dv}{dt} \qquad \text{QED.}$$

b) $acc = 24$

$$\frac{6}{t^2} - 30 = 24$$

$\times t^2$ $\quad 6 - 30t^2 = 24t^2$

$$6 = 54t^2$$

$$\frac{1}{9} = t^2$$

$$\pm\frac{1}{3} = t$$

$\therefore$ $t = 1/3$ sec.

c) $\frac{dv}{dt} = 6t^{-2} - 30$

Integrate $\quad v = \frac{6t^{-1}}{-1} - 30t = \frac{-6}{t} - 30t + c$

$t = 1/3 \quad v = 18$

$$18 = -6/_{1/3} - 30(1/3) + c$$

$$18 = -18 - 10 + c$$

$$46 = c$$

$\therefore \quad V = -\frac{6}{t} - 30t + 46$

If $V = 10$

$$10 = -\frac{6}{t} - 30t + 46$$

$$\frac{6}{t} + 30t - 36 = 0$$

$\times t \qquad 6 + 30t^2 - 36t = 0$

$\div 6 \qquad 5t^2 - 6t + 1 = 0$

$$(5t - 1)(t - 1) = 0$$

either $\qquad$ or

$t = 1/5 \qquad t = 1$

$\therefore \quad t = 1/5$ sec or $t = 1$ secs

HOW TO CHOOSE ?

We know that it accelerates initially (+ acc values) and then decelerates (- acc values)

work out acc values if you are unsure

for $t=0$ $t=0.1$ $t=0.2$ $t=0.5$ $t=0.8$ etc !!

If $\quad acc = 0$

$$\frac{6}{t^2} - 30 = 0$$

$$6 = 30t^2$$

$$\frac{1}{5} = t^2$$

$$0.548 \text{ sec} = t$$

$\therefore \quad 0$ to 0.548 sec is acceleration

> 0.548 secs it decelerates

We know $t = 1/3 \quad V = 18$ m/s

This means it needs to be slowing down

so when $V = 10$ m/s $\quad t = 1$ sec

3) $a = 3 - 4t$

a) $V = 3t - 2t^2 + c$

$t = 0 \quad V = -1$

$$-1 = 0 - 0 + c$$

$$-1 = c$$

So $\quad V = 3t - 2t^2 - 1$

b) At rest means $V = 0$

$$0 = 3t - 2t^2 - 1$$

$$2t^2 - 3t + 1 = 0$$

$$(2t - 1)(t - 1) = 0$$

$$t = \tfrac{1}{2} \quad \text{or} \quad t = 1$$

$\therefore$ At rest when $t = \frac{1}{2}$ sec and $t = 1$ sec

c) From $0 \leq t < \frac{1}{2}$ negative velocity TRY $t = 0.1, 0.2$

$t = \frac{1}{2}$ $\quad V = 0$

$\frac{1}{2} < t < 1$ positive velocity TRY $t = 0.6 \quad 0.7$ etc

$t = 1$ $\quad V = 0$

$\therefore$ Need x values when $t = \frac{1}{2}$ and $t = 1$ as they are all in same direction !!

$$x = \frac{3t^2}{2} - \frac{2t^3}{3} - 1 + c$$

Now if you imagine starting the time at the position when $t = \frac{1}{2}$ for the entire motion $\quad x = 0 \quad t = 0$

$$0 = 0 - 0 - 1 + c$$

$$1 = c \qquad \therefore \; x = \frac{3}{2}t^2 - \frac{2t^3}{3}$$

$\therefore \quad t = 1/2$

$x = \frac{3}{2}\left(\frac{1}{2}\right)^2 - \frac{2}{3}\left(\frac{1}{2}\right)^3$

$x = \frac{3}{8} - \frac{2}{24}$

$x = \frac{7}{24}$ m

$t = 1$

$x = \frac{3}{2}(1) - \frac{2}{3}(1)$

$x = \frac{9}{6} - \frac{4}{6}$

$x = \frac{5}{6}$ m

$\therefore$ Distance covered is $\frac{5}{6} - \frac{7}{24}$

$= \frac{20}{24} - \frac{7}{24}$

$= \frac{13}{24}$ m

4) $F = 15t^2 - 60t \qquad m = 5kg$

$F = ma$

a) $ma = 15t^2 - 60t$

$5a = 15t^2 - 60t$

$a = 3t^2 - 12t$

$\underline{t=2}$

$a = 3(4) - 12(2)$

$a = 12 - 24$

$a = -12 \text{ m/s}^2$

b) $v = t^3 - 6t^2 + c$

$v = 35 \quad t = 0$

$35 = 0 - 0 + c$

$35 = c$

$\therefore \quad v = t^3 - 6t^2 + 35$

c) Use stationary values to find minimum speed

~~$\frac{dv}{dt} = 3t^2 - 12t = 0$ at SP's~~

~~$t^2 - 4t = 0$~~

~~$t(t-4) = 0$~~

$t = 0$, ~~$t = 4$~~

$$\frac{dv}{dt} = 3t^2 - 12t = 0 \quad \text{at} \quad SP's$$

$$t^2 - 4t = 0$$
$$t(t-4) = 0$$
$$t = 0 \quad \text{or} \quad t = 4$$

$$\frac{d^2v}{dt^2} = 6t - 12$$

$\underline{t=0}$ $\qquad$ $\frac{d^2v}{dt^2} = -12$ $\qquad$ MAX SPEED

$\underline{t=4}$ $\qquad$ $\frac{d^2v}{dt^2} = +12$ $\qquad$ MIN SPEED

$\therefore$ Least value of speed

$$V = 4^3 - 6(4^2) + 35$$
$$V = 64 - 96 + 35$$
$$V = 3 \text{ m/s.}$$

d) ~~[illegible]~~

$$v = t^3 - 6t^2 + 35$$

Integrate $\quad x = \frac{t^4}{4} - 2t^3 + 35t + c$

$t=0 \quad x=0 \qquad 0 = 0 - 0 + 0 + c$

$$0 = c \qquad \therefore x = \frac{t^4}{4} - 2t^3 + 35t$$

$\underline{t=2} \qquad x = 4 - 16 + 70 = +58 \text{ M}$

$\underline{t=8} \qquad x = 1024 - 1024 + 280 = +280 \text{m}$

$\therefore$ Distance needed $= 280 - 58$
$= 222 \text{ m}$