Integration By Parts 1 : ANSWERS

1) $\int x \ln x \, dx$

$= \int (\ln x) \times x \, dx$

$u = \ln x \qquad \frac{dv}{dx} = x$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{x^2}{2}$

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx}$$

$$= \frac{x^2}{2} \ln x - \int \frac{x^{\cancel{2}}}{2} \times \frac{1}{\cancel{x}_1} \, dx$$

$$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$$

2) $\int (3x+2) \cos x \, dx$

$u = 3x+2 \qquad \frac{dv}{dx} = \cos x$

$\frac{du}{dx} = 3 \qquad v = -\sin x$

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$$= -(3x+2) \sin x + 3 \int \sin x \, dx$$

$$= -(3x+2) \sin x - 3 \cos x + C$$

3) $\int x \sin 2x \, dx$

$u = x \qquad \frac{dv}{dx} = \sin 2x$

$\frac{du}{dx} = 1 \qquad v = -\frac{\cos 2x}{2}$

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$$= -\frac{x \cos 2x}{2} - \int -\frac{\cos 2x}{2} \, dx$$

$$= -\frac{x \cos 2x}{2} + \frac{1}{2} \frac{\sin 2x}{2} + c$$

$$= \frac{\sin 2x}{4} - \frac{x \cos 2x}{2} + C$$

(4) $\int (3x+1)e^{2x}\,dx$

$u = 3x+1$ $\quad \frac{du}{dx} = 3$ $\quad \frac{dv}{dx} = e^{2x}$ $\quad v = \frac{e^{2x}}{2}$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$= \frac{(3x+1)e^{2x}}{2} - \frac{1}{2}\int 3e^{2x}\,dx$$

$$= \frac{(3x+1)e^{2x}}{2} - \frac{3}{2}\frac{e^{2x}}{2} + C$$

$$= \frac{(3x+1)e^{2x}}{2} - \frac{3}{4}e^{2x} + C$$

$$= \frac{e^{2x}}{2}\left(3x+1-\frac{3}{2}\right) + C$$

$$= \frac{e^{2x}}{2}\left(3x-\frac{1}{2}\right) + C$$

$$= \frac{e^{2x}}{2}\frac{(6x-1)}{2}$$

$$= \frac{e^{2x}}{4}(6x-1) + C$$

(5) $\int (x+3)e^{2x}\,dx$

$u = x+3$ $\quad \frac{du}{dx} = 1$ $\quad \frac{dv}{dx} = e^{2x}$ $\quad v = \frac{e^{2x}}{2}$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$= \frac{(x+3)e^{2x}}{2} - \frac{1}{2}\int e^{2x}\times 1\,dx$$

$$= \frac{(x+3)e^{2x}}{2} - \frac{e^{2x}}{4} + C$$

$$= \frac{e^{2x}}{2}\left(x+3-\frac{1}{2}\right) + C$$

$$= \frac{e^{2x}}{2}\left(x+\frac{5}{2}\right) + C$$

$$= \frac{e^{2x}}{2}\left(\frac{2x+5}{2}\right) + C$$

$$= \frac{e^{2x}(2x+5)}{4} + C$$

6) a) Let $I = \int x \cos 2x \, dx$

$$u = x \qquad \frac{dv}{dx} = \cos 2x$$

$$\frac{du}{dx} = 1 \qquad v = \frac{\sin 2x}{2}$$

$$I = \int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$$= \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} \times 1 \, dx$$

$$= \frac{x \sin 2x}{2} - \left(-\frac{\cos 2x}{4} \right) + C$$

$$= \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + C$$

b) $\int x \cos^2 x \, dx$

$\cancel{\int u \frac{dv}{dx}}$ First

$$\cos 2x = 2\cos^2 x - 1$$

$$\frac{\cos 2x + 1}{2} = \cos^2 x$$

$$= \int x \left(\frac{\cos 2x + 1}{2} \right) dx$$

$$= \frac{1}{2} \int x (\cos 2x + 1) \, dx$$

$$= \frac{1}{2} \int x \cos 2x + x \, dx$$

$$= \frac{1}{2} \underbrace{\int x \cos 2x \, dx}_{\text{part (a)}} + \frac{1}{2} \int x \, dx$$

$$= \frac{1}{2} \left[\frac{x \sin 2x}{2} + \frac{\cos 2x}{4} \right] + \frac{x^2}{4} + C$$

$$= \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + \frac{x^2}{4} + C$$