Integration By Parts 3 : ANSWERS

1) $\int x \sin x \, dx$

$u = x \qquad \frac{dv}{dx} = \sin x$

$\frac{du}{dx} = 1 \qquad v = -\cos x$

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$$= -x\cos x - \int -\cos x (1) \, dx$$

$$= -x\cos x + \sin x + C$$

2) $\int 2x \sin(3x-1) \, dx$

$u = x \qquad \frac{dv}{dx} = \sin(3x-1)$

$\frac{du}{dx} = 1 \qquad v = -\frac{\cos(3x-1)}{3}$

$$= 2\int x \sin(3x-1) \, dx$$

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$$= 2\left[-\frac{x\cos(3x-1)}{3} - \int -\frac{\cos(3x-1)}{3} \, dx \right]$$

$$= 2\left[-\frac{x\cos(3x-1)}{3} + \frac{\sin(3x-1)}{9} \right] + C$$

$$= \frac{2}{9}\sin(3x-1) - \frac{2x}{3}\cos(3x-1) + C$$

3) $\int x^2 \ln x \, dx$

$u = \ln x \qquad \frac{dv}{dx} = x^2$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{x^3}{3}$

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3} \times \frac{1}{x} \, dx$$

$$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 \, dx$$

$$= \frac{x^3}{3}\ln x - \frac{1}{3} \times \frac{x^3}{3} + C$$

$$= \frac{x^3}{3}\left(\ln x - \frac{1}{3} \right) + C$$

$$= \frac{x^3}{3}\left(\frac{3\ln x - 1}{3} \right) + C$$

$$= \frac{x^3}{9}(3\ln x - 1) + C$$

4) $\int (x+1)\ln x \, dx$

$u = \ln x \quad \frac{dv}{dx} = x+1$

$\frac{du}{dx} = \frac{1}{x}$

$V = \frac{x^2}{2} + x$

$V = x\left(\frac{x}{2}+1\right)$

$V = x\left(\frac{x+2}{2}\right)$

$V = \frac{x}{2}(x+2)$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$= \frac{x}{2}(x+2)\ln x - \int \frac{\cancel{x}}{2}(x+2)\frac{1}{\cancel{x}}\,dx$$

$$= \frac{x}{2}(x+2)\ln x - \frac{1}{2}\left(\frac{x^2}{2}+2x\right) + C$$

$$= \frac{x}{2}(x+2)\ln x - \frac{1}{2}\left(\frac{x^2+4x}{2}\right) + C$$

$$= \frac{x}{2}(x+2)\ln x - \frac{x}{4}(x+4) + C$$

5) $\int x e^x \, dx$

$u = x \quad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 1 \quad V = e^x$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$= xe^x - \int e^x \times 1 \, dx$$

$$= xe^x - e^x + C$$

$$= e^x(x-1) + C$$

6) $\int x^2 e^{3x} \, dx$

$u = x^2 \quad \frac{dv}{dx} = e^{3x}$

$\frac{du}{dx} = 2x \quad V = \frac{e^{3x}}{3}$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\int xe^{3x}\,dx$$

$$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[\frac{xe^{3x}}{3} - \frac{1}{3}\int e^{3x}\,dx\right]$$

$u = x \quad \frac{dv}{dx} = e^{3x}$

$\frac{du}{dx} = 1 \quad V = \frac{e^{3x}}{3}$

$$= \frac{x^2e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{3\times3\times3} + C$$

$$= \frac{x^2e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + C$$

$$= \frac{e^{3x}}{27}\left(9x^2 - 6x + 2\right) + C$$

7) $\int e^x \cos 2x \, dx$

$u = e^x \qquad \frac{dv}{dx} = \cos 2x$

$\frac{du}{dx} = e^x \qquad v = \frac{\sin 2x}{2}$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$= \frac{e^x \sin 2x}{2} - \frac{1}{2}\int e^x \sin 2x \, dx$$

$u = e^x \qquad \frac{dv}{dx} = \sin 2x$

$\frac{du}{dx} = e^x \qquad v = -\frac{\cos 2x}{2}$

$$= \frac{e^x \sin 2x}{2} - \frac{1}{2}\left[-\frac{e^x \cos 2x}{2} + \int \frac{e^x \cos 2x}{2} dx\right]$$

$$\int e^x \cos 2x \, dx = \frac{e^x \sin 2x}{2} + \frac{e^x \cos 2x}{4} - \int \frac{e^x \cos 2x}{4} dx$$

$$\frac{5}{4}\int e^x \cos 2x \, dx = \frac{e^x \sin 2x}{2} + \frac{e^x \cos 2x}{4}$$

$$\int e^x \cos 2x \, dx = \frac{4}{5}\left[\frac{e^x \sin 2x}{2} + \frac{e^x \cos 2x}{4}\right]$$

$$= \frac{2e^x}{5}\left[\sin 2x + \frac{\cos 2x}{2}\right]$$

$$= \frac{2e^x}{5}\left[\frac{2\sin 2x + \cos 2x}{2}\right]$$

$$= \frac{2e^x}{5 \times 2}\left[2\sin 2x + \cos 2x\right] + C$$

$$= \frac{e^x}{5}\left(2\sin 2x + \cos 2x\right) + C$$

8) $\int e^x \sin x \, dx$

$u = e^x$ $\qquad$ $\frac{dv}{dx} = \sin x$

$\frac{du}{dx} = e^x$ $\qquad$ $v = -\cos x$

$$\int u\frac{dv}{dx}\, dx = uv - \int v\frac{du}{dx}\, dx$$

$$= -e^x\cos x - \int -e^x \cos x \, dx$$

$u = e^x$ $\qquad$ $\frac{dv}{dx} = \cos x$

$\frac{du}{dx} = e^x$ $\qquad$ $v = \sin x$

$$= -e^x\cos x + \int e^x \cos x \, dx$$

$$= -e^x\cos x + \left[e^x \sin x - \int e^x \sin x \, dx \right]$$

$$\int e^x \sin x \, dx = -e^x\cos x + e^x \sin x - \int e^x \sin x \, dx$$

$$2\int e^x \sin x \, dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x \, dx = \frac{e^x}{2}(\sin x - \cos x)$$