Year 12 Dec 2020 Assessment
Mr. Hoppins : Answers

1) a) $\dfrac{(2\sqrt{3} + a)}{(\sqrt{3} - 1)} \times \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$$= \frac{6 + 2\sqrt{3} + a\sqrt{3} + a}{3 + \sqrt{3} - \sqrt{3} - 1}$$

$$= \frac{6 + a + (2+a)\sqrt{3}}{2}$$

b) $\dfrac{2\sqrt{6b^2}}{\sqrt{2}} - \sqrt{27} + \sqrt{192}$

$$= \frac{2\sqrt{6}\,b}{\sqrt{2}} - 3\sqrt{3} + 8\sqrt{3}$$

$$= \frac{2\cancel{\sqrt{2}}\sqrt{3}\,b}{\cancel{\sqrt{2}}} - 3\sqrt{3} + 8\sqrt{3}$$

$$= 2b\sqrt{3} + 5\sqrt{3}$$

$$= (5 + 2b)\sqrt{3}$$

2) $x^2 + 2kx + 9k = -4x$

$x^2 + (2k+4)x + 9k = 0$

$a = 1 \quad b = 2k+4 \quad c = 9k$

2 real distinct roots $\quad b^2 - 4ac > 0$

$$(2k+4)^2 - 4(1)(9k) > 0$$

$$4k^2 + 16k + 16 - 36k > 0$$

$$4k^2 - 20k + 16 > 0$$

$$k^2 - 5k + 4 > 0$$

$$(k-4)(k-1) > 0$$

$(+)\ (+) = (+)$

or $(-)\ (-) = (+)$

either $k-4>0$ and $k-1>0$

$k>4$ $\quad$ $k>1$

$k>4$

or $k-4<0$ and $k-1<0$

$k<4$ $\quad$ $k<1$

$k<1$

3) $12x^3-29x^2+7x+6=0$

$f(1) = 12-29+7+6 \neq 0$
$f(-1) = -12-29-7+6 \neq 0$
$f(2) = 96-116+14+6 = 0$

So $(x-2)$ is a factor

$12x^3-29x^2+7x+6 = (x-2)(ax^2+bx+c)$

Compare x^3 $\quad$ Compare const $\quad$ Compare x

$12 = a$ $\quad$ $6 = -2c$ $\quad$ $7 = c-2b$

$-3 = c$ $\quad$ $7 = -3-2b$

$2b = -10$

$b = -5$

$\therefore$ $12x^3-29x^2+7x+6 = (x-2)(12x^2-5x-3)$

$= (x-2)(4x-3)(3x+1)$

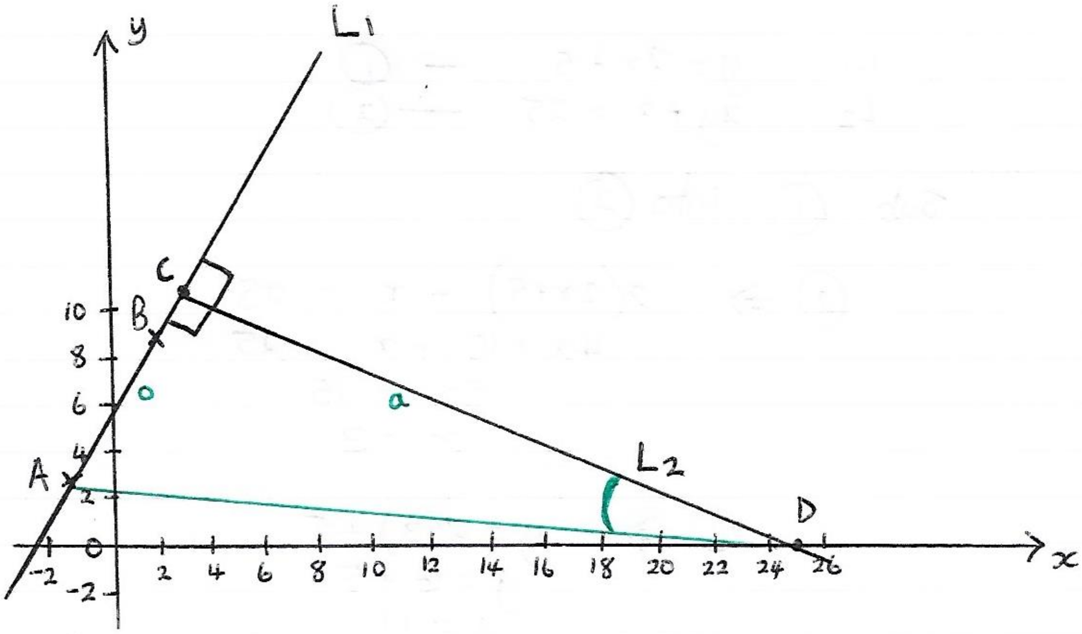
EQN is

$\therefore$ $(x-2)(4x-3)(3x+1) = 0$

either $x-2=0$ $\quad$ $4x-3=0$ $\quad$ $3x+1=0$

$x=2$ $\quad$ $x=\frac{3}{4}$ $\quad$ $x=-1/3$

4)



a) $\underline{L_1}$ Gradient $m = \dfrac{y_2 - y_1}{x_2 - x}$

$A\ (-1, 3)$ ($x_1\ y_1$)

$B\ (2, 9)$ ($x_2\ y_2$)

$= \dfrac{9-3}{2-(-1)}$

$= \dfrac{6}{3}$

$= 2$

Use $B(2,9)$ ($x_2\ y_2$)

Eqn. of L_1

$$y - y_2 = m(x - x_2)$$
$$y - 9 = 2(x-2)$$
$$y - 9 = 2x - 4$$
$$y = 2x + 5$$

b) (i) L_2 crosses x axis when $y = 0$

$$2y + x = 25$$
$$0 + x = 25$$
$$x = 25$$

$\therefore D(25, 0)$

(ii) $\underline{L_2}$

$$2y + x = 25$$
$$y + \tfrac{1}{2}x = \frac{25}{2}$$
$$y = -\tfrac{1}{2}x + \frac{25}{2}$$

$\therefore m = -\tfrac{1}{2}$

<u>Now</u>

$\underset{L_1}{2} \times \underset{L_2}{(-\tfrac{1}{2})} = -1$ $\quad\therefore$ L_1 and L_2 are perpendicular

(iii) To find C solve L_1 and L_2 eqns. simultaneously

$$L_1 \quad y = 2x+5 \quad — ①$$
$$L_2 \quad 2y + x = 25 \quad — ②$$

Sub ① into ②

$$② \Rightarrow \quad 2(2x+5) + x = 25$$
$$4x + 10 + x = 25$$
$$5x = 15$$
$$x = 3$$

$$① \Rightarrow \quad y = 2(3) + 5$$
$$y = 6 + 5$$
$$y = 11$$

$\therefore$ $C(3, 11)$

c) $C(3,11)$ $D(25,0)$

$\quad x_3 \; y_3 \qquad x_4 \; y_4$

$$CD = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$$
$$= \sqrt{(25-3)^2 + (0-11)^2}$$
$$= \sqrt{22^2 + (-11)^2}$$
$$= \sqrt{485 + 121}$$
$$= \sqrt{605} \text{ units}$$
$$= \sqrt{121 \times 5}$$
$$= 11\sqrt{5} \text{ units.}$$

d)

$$\tan \hat{ADB} = \frac{AC}{CD}$$

Find AC first

$$AC = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$= \sqrt{[3-(-1)]^2 + (11-3)^2}$$
$$= \sqrt{16 + 64}$$
$$= \sqrt{80} = 4\sqrt{5}$$

$$\therefore \tan \hat{ADB} = \frac{4\sqrt{5}}{11\sqrt{5}}$$
$$\tan \hat{ADB} = \frac{4}{11}$$
$$\hat{ADB} = 20.0^\circ$$

5) $\overset{x_1\ y_1}{P(1,-4)}$ $\quad$ $\overset{x_2\ y_2}{Q(9,10)}$

a) (i) A is mid-point of PQ

$$A\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= A\left(\frac{1+9}{2}, \frac{-4+10}{2}\right)$$

$$= A(5,3)$$

(ii) $PQ = \text{Diameter} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$= \sqrt{(9-1)^2+(10-(-4))^2}$$

$$= \sqrt{64+196}$$

$$= \sqrt{260}$$

$$= \sqrt{4}\sqrt{65}$$

$$= 2\sqrt{65}$$

$\therefore$ radius $r = \sqrt{65}$

(iii) C has equation $\quad$ centre $(5,3)$ radius $\sqrt{65}$

$$(x-5)^2+(y-3)^2 = 65$$

b) $R(4,11)$

<u>check</u> $x=4$ $y=11$ into eqn. above.

$$(4-5)^2+(11-3)^2 = 65$$

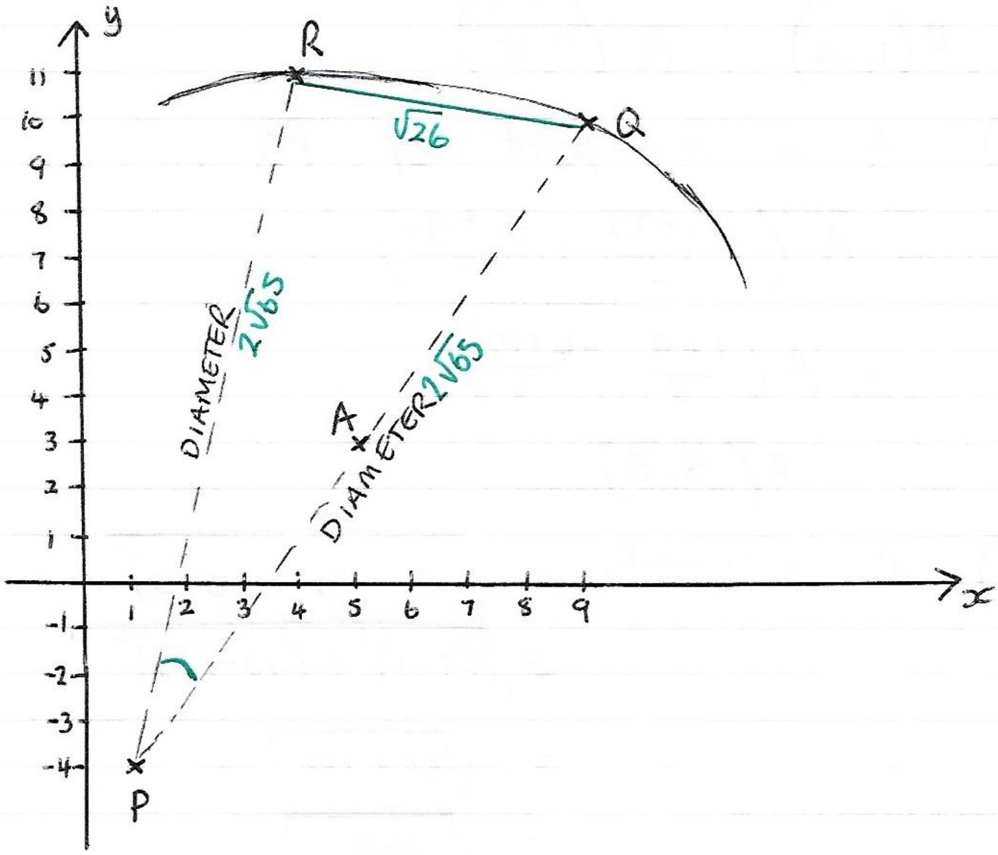
$$(-1)^2+(8)^2 = 65$$

$$1+64 = 65$$

$$65 = 65 \checkmark$$

$\therefore$ R lies on C.

c) PTO



Find $\underline{RQ}$

$$RQ = \sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$$
$$= \sqrt{(4-9)^2+(11-10)^2}$$
$$= \sqrt{25+1}$$
$$= \sqrt{26}$$

Cosine Rule

$$RQ^2 = PR^2 + PQ^2 - 2(PR)(PQ)\cos Q\hat{P}R$$
$$26 = 4(65) + 4(65) - 2(2\sqrt{65})(2\sqrt{65})\cos Q\hat{P}R$$
$$26 = 520 - 520\cos Q\hat{P}R$$
$$520\cos Q\hat{P}R = 494$$
$$\cos Q\hat{P}R = \frac{494}{520}$$
$$Q\hat{P}R = 18{\cdot}2^\circ$$

6) a) Let $y = 2x^5 + 24x^{-2} - 3x^{1/2}$

$$\frac{dy}{dx} = 10x^4 - 48x^{-3} - \frac{3}{2}x^{-1/2}$$

$$= 10x^4 - \frac{48}{x^3} - \frac{3}{2\sqrt{x}}$$

b) Let $y = x^2(3x+1)$

$$y = 3x^3 + x^2$$

$$\frac{dy}{dx} = 9x^2 + 2x$$

7) $x^2 + 4x + 9$

$$= (x+2)^2 - 4 + 9$$

$$= (x+2)^2 + 5 \qquad a = 2 \quad b = 5$$

Now $$\frac{1}{x^2+4x+9} = \frac{1}{(x+2)^2+5}$$

MAX value when $(x+2)^2+5$ is minimum or as close to ZERO as possible.

$$\text{MAX} = \frac{1}{0+5} = \frac{1}{5}$$