Year 13 Assessment Nov 2020 Paper 1

1) a) $T_8 = 576$ $T_9 = 2304$

① $- \; ar^7 = 576$ $ar^8 = 2304 \; -$ ②

② ÷ ① $\dfrac{ar^8}{ar^7} = \dfrac{2304}{576}$

$r = 4$

① $\Rightarrow$ $a(4^7) = 576$

$a = \dfrac{576}{4^7}$

$a = \dfrac{9}{256}$

$\therefore \; T_5 = ar^4$

$T_5 = \dfrac{9}{256} \times 4^4$

$T_5 = 9$

b)(i) $a = a$ $r = r$

$T_3 = 24$

① $- \; ar^2 = 24$

$a = \dfrac{24}{r^2}$

$T_2 + T_3 + T_4 = -56$

$ar + ar^2 + ar^3 = -56$

$\dfrac{24r}{r^2} + 24 + \dfrac{24}{r^2}r^3 = -56$

$\dfrac{24}{r} + 24 + 24r = -56$

$\times r$ $24 + 24r + 24r^2 = -56r$

$24r^2 + 80r + 24 = 0$

$\div 24$ $r^2 + \dfrac{10}{3}r + 1 = 0$

$\times 3$ $3r^2 + 10r + 3 = 0$

(ii) $|r| < 1$

$$S_\infty = \frac{a}{(1-r)}$$

$$3r^2 + 10r + 3 = 0$$

$$(3r+1)(r+3) = 0$$

$r = -1/3$ $\quad \cancel{r = -3}$

Now $a = \dfrac{24}{r^2}$

$a = \dfrac{24}{1/9} = 24 \times 9 = 216$

$$\therefore \quad S_\infty = \frac{216}{1-(-1/3)}$$

$$= \frac{216}{4/3}$$

$$= 216 \times \frac{3}{4}$$

$$= 54 \times 3$$

$$= 162$$

2) a) $4\cot^2\theta - 8 = 2\operatorname{cosec}^2\theta - 5\operatorname{cosec}\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$4(\operatorname{cosec}^2\theta - 1) - 8 = 2\operatorname{cosec}^2\theta - 5\operatorname{cosec}\theta$$

$$4\operatorname{cosec}^2\theta - 4 - 8 = 2\operatorname{cosec}^2\theta - 5\operatorname{cosec}\theta$$

$$2\operatorname{cosec}^2\theta + 5\operatorname{cosec}\theta - 12 = 0$$

$$(2\operatorname{cosec}\theta - 3)(\operatorname{cosec}\theta + 4) = 0$$

either

$2\operatorname{cosec}\theta - 3 = 0$

$\operatorname{cosec}\theta = 3/2$

$\sin\theta = 2/3$

$\alpha = 41.8^\circ$

Sin +ve 1st + 2nd

$\theta = 41.8^\circ, 138.2^\circ$

or

$\operatorname{cosec}\theta + 4 = 0$

$\operatorname{cosec}\theta = -4$

$\sin\theta = -1/4$

$\alpha = 14.5^\circ$

Sin −ve 3rd + 4th

$x = 194.5^\circ, 345.5^\circ$

b) $\sec\phi + 2\tan\phi = 0$

$\sec\phi = -2\tan\phi$

$\frac{1}{\cos\phi} = \frac{-2\sin\phi}{\cos\phi}$

$\times \cos\phi$ $\quad \cos\phi = -2\sin\phi\cos\phi$

$2\sin\phi\cos\phi + \cos\phi = 0$

$\cos\phi(2\sin\phi + 1) = 0$

either $\cos\phi = 0$ or $2\sin\phi + 1 = 0$

$\phi = 90°, 270°$

$\sin\phi = -\frac{1}{2}$

$\alpha = 30°$

Sin -ve 3rd + 4th

$\phi = 210°, 330°$

$\therefore \phi = 90°, 210°, 270°, 330°$

3) a) $\tan 2x = 4\tan x$

$\frac{2\tan x}{1-\tan^2 x} = 4\tan x$

$2\tan x = 4\tan x(1-\tan^2 x)$

$2\tan x = 4\tan x - 4\tan^3 x$

$\div 2$

$2\tan^3 x - \tan x = 0$

$\tan x(2\tan^2 x - 1) = 0$

either $\tan x = 0$ or $2\tan^2 x - 1 = 0$

$x = 0°, 180°, 360°$

$\tan x = \pm\frac{1}{\sqrt{2}}$

All 4 quads

$\alpha = 35.3°$

$x = 35.3°, 144.7°, 215.3°, 324.7°$

$\therefore x = 0°, 35.3°, 144.7°, 180°, 215.3°, 324.7°, 360°$

b) $7\cos\theta + 24\sin\theta = \sqrt{7^2+24^2}\left[\frac{7}{25}\cos\theta + \frac{24}{25}\sin\theta\right]$

$= 25\left[\frac{7}{25}\cos\theta + \frac{24}{25}\sin\theta\right]$

$\cos\alpha\cos\theta + \sin\alpha\sin\theta$

$= 25\cos(\theta-\alpha)$

where $\cos\alpha = 7/25$

$\alpha = 73{\cdot}7°$

$\therefore\ 7\cos\theta + 24\sin\theta = 25\cos(\theta - 73{\cdot}7°)$

NOW $7\cos\theta + 24\sin\theta = 16$

$25\cos(\theta-73{\cdot}7°) = 16$

$\cos(\theta - 73{\cdot}7°) = 16/25$

$\alpha = 50{\cdot}2°$

Cos +ve 1st + 4th quads

$\therefore\ \theta - 73{\cdot}7° = 50{\cdot}2°,\ 309{\cdot}8°$

$\theta = 123{\cdot}2°$, Too Big.

4) $x = 3\cos t$ $\qquad y = 4\sin t$

$\frac{dx}{dt} = -3\sin t$ $\qquad \frac{dy}{dt} = 4\cos t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= 4\cos t \times \frac{1}{-3\sin t}$$

$$= -\frac{4\cos t}{3\sin t}$$

a) $y - y_1 = m(x - x_1)$ at parameter p point

$$y - 4\sin p = -\frac{4}{3}\frac{\cos p}{\sin p}(x - 3\cos p)$$

$$(3\sin p)y - 12\sin^2 p = -(4\cos p)x + 12\cos^2 p$$

$$(3\sin p)y + (4\cos p)x - 12\sin^2 p - 12\cos^2 p = 0$$

$$(3\sin p)y + (4\cos p)x - 12(\sin^2 p + \cos^2 p) = 0$$

$$(3\sin p)y + (4\cos p)x - 12 = 0$$

b) Tangent crosses x axis $y = 0$

(i) $(4\cos p)x - 12 = 0$

$$x = \frac{12}{4\cos p}$$

$p = \pi/6$ $\qquad x = \frac{12}{4\cos \pi/6} = \frac{3}{\sqrt{3}/2} = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

$\therefore$ $A(2\sqrt{3}, 0)$
$\qquad\qquad x_1 \quad y_1$

Tangent crosses y axis $x = 0$

$(3\sin p)y - 12 = 0$

$$y = \frac{12}{3\sin p}$$

$p = \pi/6$

$$y = \frac{12}{3\sin\pi/6}$$

$$y = \frac{4}{\sin\pi/6} = 8$$

$$\therefore \; B(0,8)$$

$$\qquad x_2 \; y_2$$

(ii)

$$AB = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$

$$= \sqrt{(8-0)^2+(0-2\sqrt{3})^2}$$

$$= \sqrt{64+12}$$

$$= \sqrt{76}$$

$$= 2\sqrt{19}$$

5) $$\left(1+\frac{x}{3}\right)^{-1/2} = 1 + \binom{-\frac{1}{2}}{1}1\left(\frac{x}{3}\right) + \binom{-\frac{1}{2}}{2}1\left(\frac{x}{3}\right)^2 + \cdots$$

$$= 1 - \frac{x}{6} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2\times 1}\frac{x^2}{9} + \cdots$$

$$= 1 - \frac{x}{6} + \frac{x^2}{24} - \cdots$$

$1 + \frac{x}{3} = 0$ Valid for $|x| < 3$

$\frac{x}{3} = -1$

$x = -3$

$x = \frac{1}{5}$ $$\left(1+\frac{1}{15}\right)^{-1/2} = 1 - \frac{\left(\frac{1}{5}\right)}{6} + \frac{\left(\frac{1}{5}\right)^2}{24} - \cdots$$

$$\frac{1}{\sqrt{\frac{16}{15}}} = 1 - \frac{1}{30} + \frac{1}{600}$$

$$\frac{\sqrt{15}}{4} = \frac{600-20+1}{600} = \frac{581}{600}$$

$$\sqrt{15} = \frac{581}{150} \qquad a = 581 \quad b = 150$$

6) a) $a = 4 \qquad d = 6$

(i) $T_n = a + (n-1)d$

$T_n = 4 + (n-1)6$

$T_n = 4 + 6n - 6$

$T_n = 6n - 2$

(ii) $S_n = 4 + 10 + \dots + (6n-8) + (6n-2)$

$S_n = (6n-2) + (6n-8) + \dots + 10 + 4$

ADD $2S_n = \underbrace{(6n+2) + (6n+2) + \dots + (6n+2) + (6n+2)}_{n \text{ lots}}$

$2S_n = n(6n+2)$

$\div 2$ $S_n = n(3n+1)$

b) $T_{10} = 4T_5 \qquad S_{15} = 210$

(i) $a + 9d = 4(a + 4d)$

① — $a + 9d = 4a + 16d$

$\frac{15}{2}[2a + 14d] = 210$

$15(2a + 14d) = 420$

$2a + 14d = 28$

$a + 7d = 14$

$a = 14 - 7d$ — ②

① ⇒ $14 - 7d + 9d = 4(14 - 7d) + 16d$

$14 + 2d = 56 - 28d + 16d$

$14d = 56 - 14$

$14d = 42$

$d = 3$

$a = 14 - 7(3)$

$a = -7$

(ii) $T_k = 200$

$a + (k-1)d = 200$

$-7 + (k-1)3 = 200$

$-7 + 3k - 3 = 200$

$3k = 210$

$k = 70$