(a) The eighth and ninth terms of a geometric series are 576 and 2304 respectively. Find the fifth term of the geometric series. [3]

(b) Another geometric series has first term a and common ratio r. The third term of this geometric series is 24. The sum of the second, third and fourth terms of the series is -56.

(i) Show that r satisfies the equation

$$3r^2 + 10r + 3 = 0.$$

(ii) Given that $|r|<1$, find the value of r and the sum to infinity of the series. [8]

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2. (a) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$4\cot^2\theta - 8 = 2\operatorname{cosec}^2\theta - 5\operatorname{cosec}\theta.$$ [6]

(b) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

$$\sec\phi + 2\tan\phi = 0.$$ [3]

3. (a) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan 2x = 4\tan x.$$ [5]

(b) Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.
Hence, find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$7\cos\theta + 24\sin\theta = 16.$$ [6]

4. The curve C has the parametric equations

$$x = 3\cos t,\ y = 4\sin t.$$

The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$(3\sin p)y + (4\cos p)x - 12 = 0.$$ [5]

(b) The tangent to C at the point P meets the x-axis at the point A and the y-axis at the point B. Given that $p = \frac{\pi}{6}$,

(i) find the coordinates of A and B,

(ii) show that the exact length of AB is $2\sqrt{19}$. [4]

5. Expand $\left(1+\frac{x}{3}\right)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2.

State the range of values of x for which your expansion is valid.

Hence, by writing $x = \frac{1}{5}$ in your expansion, find an approximate value for $\sqrt{15}$ in the form $\frac{a}{b}$, where a and b are integers whose values are to be found. [5]

6. (a) The first term of an arithmetic series is 4 and the common difference is 6.

(i) Show that the nth term of the arithmetic series is $6n-2$.

(ii) The sum of the first n terms of this series is given by

$$S_n = 4 + 10 + \ldots + (6n-8) + (6n-2).$$

Without using the formula for the sum of the first n terms of an arithmetic series, **prove** that

$$S_n = n(3n+1).$$ [4]

(b) The tenth term of another arithmetic series is four times the fifth term. The sum of the first fifteen terms of the series is 210.

(i) Find the first term and common difference of this arithmetic series.

(ii) Given that the kth term of the series is 200, find the value of k.

[6]