1. Given that $f(x) = \dfrac{x^2 + x + 13}{(x+2)^2(x-3)}$,

   (a) express $f(x)$ in terms of partial fractions, [4]

   (b) evaluate

   $$\int_6^7 f(x)\,dx,$$

   giving your answer correct to three decimal places. [3]

2. Find the equation of the normal to the curve

   $$x^4 - 2x^2y + y^2 = 4$$

   at the point $(1, 3)$. [5]

3. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

   (a) $(7 - 9x^2)^5$ (b) $\tan^{-1} 6x$ [2], [2]

   (c) $e^{4x}\tan 2x$ (d) $\dfrac{3 + \sin x}{2 + \cos x}$ [3], [3]

4. (a) Find

   (i) $\displaystyle\int \cos\left(3x + \frac{\pi}{2}\right) dx,$ (ii) $\displaystyle\int e^{3-4x}\,dx,$

   (iii) $\displaystyle\int \frac{7}{8x+5}\,dx.$ [6]

   (b) Evaluate $\displaystyle\int_1^2 \frac{9}{(2x-1)^4}\,dx.$ [4]

5. The size N of the population of a small island may be modelled as a continuous variable. At time t, the rate of increase of N is directly proportional to the value of N.

   (a) Write down the differential equation that is satisfied by N. [1]

   (b) Show that $N = Ae^{kt}$, where A and k are constants. [3]

   (c) Given that $N = 100$ when $t = 2$ and that $N = 160$ when $t = 12$,

        (i) show that $k = 0.047$, correct to three decimal places,

        (ii) find the size of the population when $t = 20$. [7]

6. (a) Find $\int xe^{-2x}\,dx$. [4]

(b) Use the substitution $u = 1 + 3\ln x$ to evaluate

$$\int_1^e \frac{1}{x(1+3\ln x)}\,dx.$$

Give your answer correct to four decimal places. [4]

7. Water is leaking from a hole at the bottom of a large tank. The volume of the water in the tank at time t hours is $V\,\text{m}^3$. The rate of decrease of V is directly proportional to V^3.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that $V = 60$ when $t = 0$, show that

$$V^2 = \frac{3600}{at+1},$$

where a is a constant. [4]

(c) When $t = 2$, the volume of the water in the tank is $50\,\text{m}^3$. Find the value of t when the volume of the water in the tank is $27\,\text{m}^3$. Give your answer correct to one decimal place. [4]