

Trigonometry 9: Small Angles : ANSWERS

i) a) $\lim_{\theta \rightarrow 0} \left(\frac{\sin 3\theta + \tan 5\theta}{2\theta} \right)$

$$= \frac{3\theta + 5\theta}{2\theta}$$

$$= \frac{8\theta}{2\theta}$$

$$= 4$$

b) $\lim_{\theta \rightarrow 0} \left(\frac{\cos 2\theta - 1}{\theta \sin 5\theta} \right)$

$$= \frac{1 - \frac{1}{2}(2\theta)^2 - 1}{\theta \times 5\theta}$$

$$= \frac{-4\theta^2}{10\theta^2}$$

$$= -\frac{2}{5}$$

c) $\lim_{\theta \rightarrow 0} \left(\frac{\sin 2\theta}{\theta} \right)$

$$= \frac{2\theta}{\theta}$$

$$= 2$$

$$\begin{aligned}
 2) \quad a) \quad & \frac{\cos \theta - 1}{\sin \theta} \\
 & \approx \frac{1 - \frac{\theta^2}{2} - 1}{\theta} \\
 & \approx \frac{-\frac{\theta^2}{2}}{\theta} \\
 & \approx -\frac{\theta}{2}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{\sin \theta}{1 - \cos 2\theta} \\
 & \approx \frac{\theta}{1 - (1 - \frac{(2\theta)^2}{2})} \\
 & \approx \frac{\theta}{1 - 1 + \frac{4\theta^2}{2}} \\
 & \approx \frac{\theta}{2\theta^2} \\
 & \approx \frac{1}{2\theta}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \frac{\sin 3\theta + \tan \theta}{\cos 2\theta} \\
 & \approx \frac{3\theta + \theta}{1 - \frac{(2\theta)^2}{2}} \\
 & \approx \frac{4\theta}{1 - 2\theta^2}
 \end{aligned}$$

$$c) \quad \frac{21 + 7\tan \theta - 20\cos \theta}{1 + \sin 2\theta}$$

$$\approx \frac{21 + 7\theta - 20(1 - \frac{1}{2}\theta^2)}{1 + 2\theta}$$

$$\approx \frac{21 + 7\theta - 20 + 10\theta^2}{1 + 2\theta}$$

$$\approx \frac{10\theta^2 + 7\theta + 1}{1 + 2\theta}$$

$$\approx \frac{(5\theta + 1)(2\theta + 1)}{(1 + 2\theta)}$$

$$\div (1 + 2\theta)$$

$$\div (1 + 2\theta)$$

$$\approx 5\theta + 1$$

$$d) \frac{1 + \sin \theta}{5 + 3 \tan \theta - 4 \cos \theta}$$

$$\approx \frac{1 + \theta}{5 + 3\theta - 4\left(1 - \frac{\theta^2}{2}\right)}$$

$$\approx \frac{1 + \theta}{5 + 3\theta - 4 + 2\theta^2}$$

$$\approx \frac{1 + \theta}{2\theta^2 + 3\theta + 1}$$

$$\approx \frac{1 + \theta}{(2\theta + 1)(\theta + 1)}$$

$$\approx \frac{1}{2\theta + 1}$$

$$3) a) \cos x - 4 \sin x = x^2$$

$$1 - \frac{x^2}{2} - 4x = x^2$$

$$\times 2 \quad 2 - x^2 - 8x = 2x^2$$

$$0 = 3x^2 + 8x - 2$$

$$0 = (3x - 1)(x + 2) \quad \times \text{ Doesn't factorise}$$

$$x = \frac{-8 \pm \sqrt{64 - 4(3)(-2)}}{6}$$

$$x = \frac{-8 \pm \sqrt{88}}{6}$$

$$x = \frac{-8 \pm 9.38}{6}$$

$$x = \frac{1.38}{6}$$

$$x = 0.23 \text{ rads.}$$

IGNORE

NEGATIVE ANSWERS

$$b) \quad \frac{21 + 7\tan x - 20\cos x}{1 + \sin 2x} = 2$$

from 2c)

$$5\theta + 1 = 2$$

$$5\theta = 1$$

$$\theta = \frac{1}{5} \text{ rad.}$$