Quadratic Equations and logs : Answers

~~Simultaneous Equations~~

(i) $4^x + 2(2^x) - 8 = 0$

Let $y = 2^x$

$(2^2)^x + 2(2^x) - 8 = 0$

$(2^x)^2 + 2(2^x) - 8 = 0$

$y^2 + 2y - 8 = 0$

$(y+4)(y-2) = 0$

either $y = -4$ or $y = 2$

$2^x \neq -4$ IMPOSSIBLE

$2^x = 2$

$x = 1$

DONE IN YOUR HEAD !!

(2) $9^x - 5(3^x) + 6 = 0$

$(3^2)^x - 5(3^x) + 6 = 0$

$(3^x)^2 - 5(3^x) + 6 = 0$

Let $y = 3^x$

$y^2 - 5y + 6 = 0$

$(y-3)(y-2) = 0$

either $y - 3 = 0$ or $y - 2 = 0$

$y = 3$ $y = 2$

$\therefore 3^x = 3$ or $3^x = 2$

$x = 1$

$\ln 3^x = \ln 2$

$x \ln 3 = \ln 2$

$x = \frac{\ln 2}{\ln 3}$

$x = 0.63$

③ $25^x + 3 = 4 \times 5^x$

$(5^2)^x + 3 = 4 \times 5^x$

$(5^x)^2 + 3 = 4 \times 5^x$

Let $y = 5^x$

$y^2 + 3 = 4y$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

either $y = 3$ or $y = 1$

$5^x = 3$

$\ln 5^x = \ln 3$

$x \ln 5 = \ln 3$

$x = \frac{\ln 3}{\ln 5}$

$x = 0.68$

$5^x = 1$

$x = 0$ ← OBVIOUS

④ $49^x = 7 \times 7^x - 10$

$(7^2)^x = 7 \times 7^x - 10$

$(7^x)^2 = 7 \times 7^x - 10$

Let $y = 7^x$

$y^2 = 7y - 10$

$y^2 - 7y + 10 = 0$

$(y-5)(y-2) = 0$

either $y = 5$ or $y = 2$

$7^x = 5$

$\ln 7^x = \ln 5$

$x \ln 7 = \ln 5$

$x = \frac{\ln 5}{\ln 7}$

$x = 0.83$

$7^x = 2$

$\ln 7^x = \ln 2$

$x \ln 7 = \ln 2$

$x = \frac{\ln 2}{\ln 7}$

$x = 0.36$

(5) $2(4^x) + 5 = 11(2^x)$

$2[(2^2)^x] + 5 = 11(2^x)$

$2[(2^x)^2] + 5 = 11(2^x)$

Let $y = 2^x$

$2y^2 + 5 = 11y$

$2y^2 - 11y + 5 = 0$

$(2y - 1)(y - 5) = 0$

either $2y - 1 = 0$ or $y - 5 = 0$

$y = \frac{1}{2}$ $y = 5$

$2^x = \frac{1}{2}$ $2^x = 5$

IN YOUR HEAD! $x = -1$ $\ln 2^x = \ln 5$

$x \ln 2 = \ln 5$

$x = \frac{\ln 5}{\ln 2}$

$x = 2.32$