1. Simplify $\dfrac{3\sqrt{3}-2\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$. [4]

2. The points A and B have coordinates $(-2, 10)$ and $(12, 3)$ respectively.

(a) (i) Find the gradient of AB.

(ii) Find the equation of AB. [4]

(b) The line L is perpendicular to AB and intersects the y-axis at the point $C(0, -1)$. The lines AB and L intersect at the point D.

(i) Write down the equation of L.

(ii) Show that D has coordinates $(4, 7)$.

(iii) Find the length of AD and the length of BD. [7]

(c) The line CD is extended to the point E so that D is the mid-point of CE.

(i) Find the coordinates of E.

(ii) **Write down** the geometrical name for the quadrilateral $ACBE$. [3]

3. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$6\sin^2\theta + 1 = 2(\cos^2\theta - \sin\theta).$$ [6]

(b) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan(3x - 57°) = -0{\cdot}81.$$ [4]

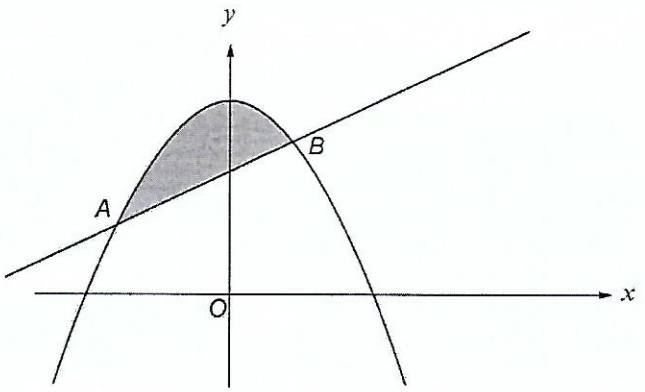
(c) Without carrying out any calculations, explain why there are no values of ϕ which satisfy the equation

$$2\sin\phi + 4\cos\phi = -7.$$ [1]

4. Given that $y = 4\sqrt{x} + \dfrac{45}{x}$, find the value of $\dfrac{dy}{dx}$ when $x = 9$. [4]

5. Find $\displaystyle\int\left(\frac{5}{x^3} - 2x^{\frac{1}{3}} - 4\right)dx$. [3]

6.



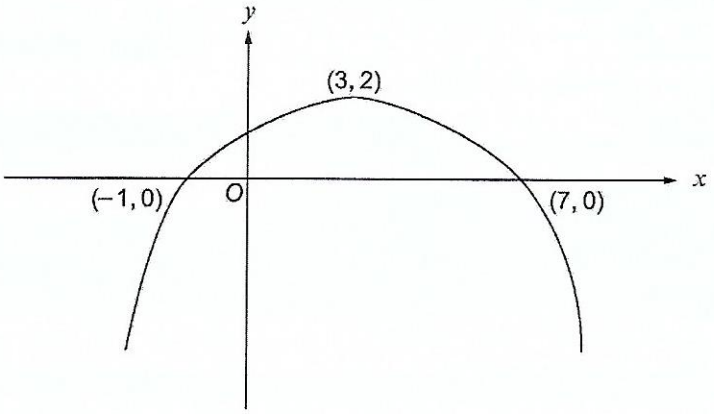
The diagram shows a sketch of the curve $y = 16 - x^2$ and the line $y = x + 10$. The line and the curve intersect at the points A and B.

(i) Find the coordinates of A and B.

(ii) Find the area of the shaded region. [10]

7.

The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-1, 0)$ and $(7, 0)$ and has a maximum point at $(3, 2)$.



(a) Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

(i) $y = f(x + 4)$

(ii) $y = -2f(x)$ [6]

(b) Hence write down one root of the equation

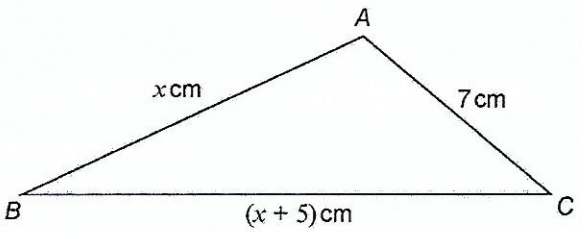
$$f(x + 4) = -2f(x) + 4.$$ [1]

8.

(a) Given that $x - 3$ is a factor of $px^3 - 13x^2 - 19x + 12$, write down an equation satisfied by p. Hence show that $p = 6$. [2]

(b) Solve the equation $6x^3 - 13x^2 - 19x + 12 = 0$. [4]

9. The diagram below shows a sketch of the triangle ABC with $AB = x$ cm, $BC = (x + 5)$ cm, $AC = 7$ cm and $\cos B\hat{A}C = -\frac{3}{5}$.



(a) Write down an equation satisfied by x. Hence show that $x = 15$. [3]

(b) Find the exact value of the area of triangle ABC. [3]

(c) The point D lies on BC and is such that AD is perpendicular to BC. Find the length of AD. [2]

10. *(a)* **Use the binomial theorem** to express $(1+\sqrt{6})^5$ in the form $a+b\sqrt{6}$, where a, b are integers whose values are to be found. [5]

(b) The coefficient of x^2 in the expansion of $(1+3x)^n$ is 495. Given that n is a positive integer, find the value of n. [3]

11. Find the values of m for which the equation $4x^2 + 8x - 8 = m(4x - 3)$ has real roots. [5]

12. The curve C has equation

$$y = -2x^3 + 12x^2 - 18x + 5.$$

(a) Find the coordinates and the nature of each of the stationary points of C. [6]

(b) Sketch C, indicating the coordinates of each of the stationary points. [2]

(c) Given that the equation

$$-2x^3 + 12x^2 - 18x + 5 = k$$

has three distinct real roots, find the range of possible values for k. [2]

13. The curve C has equation $y = \frac{20}{x} + 2x^2 - 11$. The point P has coordinates (2, 7) and lies on C.

Find the equation of the **normal** to C at P. [6]

14. (a) Solve the equation

$$4^{3x+1} = 22.$$

Show your working and give your answer correct to two decimal places. [3]

(b) Given that

$$\log_d z = 2\log_d 6 - \log_d 9 - 1,$$

express z in terms of d, giving your answer in a form **not** involving logarithms. [4]

(c) Solve the equation

$$25^x - 4 \times 5^x + 3 = 0 \quad \text{where} \quad x > 0$$ [4]

15. The circle C has centre A and radius r. The points $P(-2, -3)$ and $Q(8, 1)$ are at opposite ends of a diameter of C.

(a) (i) Write down the coordinates of A.

(ii) Show that $r = \sqrt{29}$. [3]

(b) Given that the point $R(5, 4)$ lies on the circle C, find $P\hat{Q}R$. Give your answer in degrees, correct to one decimal place. [3]

(c) The point S lies on the circle C. The tangent to the circle at S passes through the point $T(11, 0)$. Find the length of ST. [3]

16. A population, P, of a town is growing exponentially and can be modelled using the formula $P = Ae^{kt}$, where t is measured in years.

(a) If $P = 15\,000$ at the start of year 2000, and P has grown to 17 000 at the start of year 2003, find the formula for P.

(b) Based on this model, at the end of which year will the population reach 20 000? [5]

17. a) Show by counter example, that the statement

If $|a+1| = |b+1|$, then $a = b$ [3]

is false

b) Use proof by deduction to prove that if a and b are positive real numbers

$a^2 + b^2 \geq 2ab$ [3]

18. The vectors $\underline{a} = 3\underline{i} + 2\underline{j}$ and $\underline{b} = 6\underline{i} - 3\underline{j}$ are the position vectors of A and B respectively. [5]

(a) Find $\vec{AB}$

(b) Point P divides the line AB in the ratio 3:1. Find the position vector $\vec{OP}$.