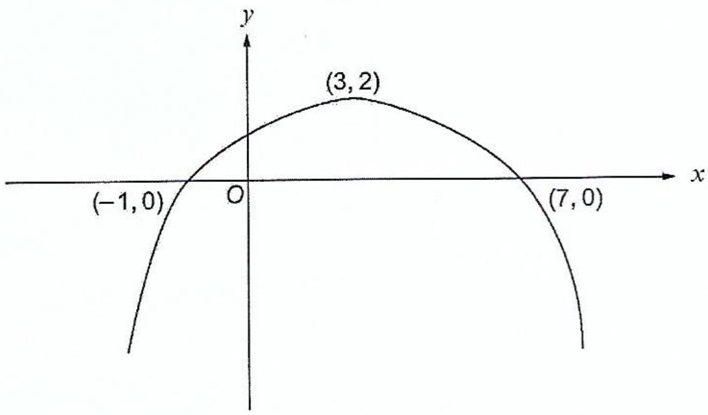
1.

Solve the equation

$$|3x + 4| = 2|x - 3|.$$ [3]

2.

The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points (–1, 0) and (7, 0) and has a maximum point at (3, 2).



(a) Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

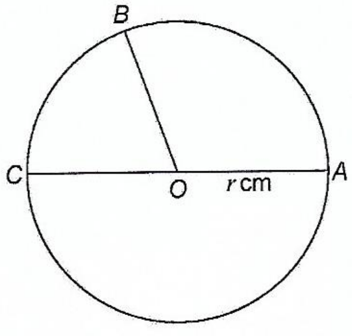
(i) $y = f(x + 4)$

(ii) $y = -2f(x)$ [6]

(b) Hence write down one root of the equation

$$f(x + 4) = -2f(x) + 4.$$ [1]

3.



The diagram shows a sketch of a circle with centre O and radius r cm. Three points A, B and C lie on the circle. The line AC is a diameter of the circle and $A\hat{O}B = 2{\cdot}15$ radians.

Given that the area of sector BOC is 26 cm^2 less than the area of sector AOB, find the value of r. Give your answer correct to one decimal place. [5]

4.

[redacted] Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$15\operatorname{cosec}^2\theta + 2\cot\theta = 23.$$ [6]

5.

[redacted] a) Express $\dfrac{9}{(x-1)(x+2)^2}$ in terms of partial fractions. [4]

b) Find $\displaystyle\int \frac{9}{(x-1)(x+2)^2}\,\mathrm{d}x$. [3]

6.

Expand $\left(1+\frac{x}{8}\right)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2.

State the range of values of x for which your expansion is valid.
Hence, by writing $x = 1$ in your expansion, find an approximate value for $\sqrt{2}$ in the form $\frac{a}{b}$, where a and b are integers whose values are to be found. [5]

7.

(a) An arithmetic series has first term a and common difference d.
Prove that the sum of the first n terms of the series is given by

$$S_n = \tfrac{n}{2}[2a + (n-1)d].$$ [3]

(b) The first term of an arithmetic series is 3 and the common difference is 2. The sum of the first n terms of the series is 360.
Write down an equation satisfied by n. Hence find the value of n. [3]

(c) The tenth term of another arithmetic series is seven times the third term. The sum of the eighth and ninth terms of the series is 80. Find the first term and common difference of this arithmetic series. [4]

8.

A geometric series has first term a and common ratio r. The sum of the second and third terms of the series is -216. The sum of the fifth and sixth terms of the series is 8.

(a) Prove that $r = -\frac{1}{3}$. [5]

(b) Find the sum to infinity of the series. [3]

9.

A function is defined parametrically by

$$x = 4\sin 3t,\ y = 2\cos 3t.$$

(a) Find and simplify an expression for $\frac{dy}{dx}$ in terms of t. [4]

(b) Find and simplify an expression for $\frac{d^2y}{dx^2}$

(i) in terms of t,

(ii) in terms of y. [4]

10.

Complete the following proof by contradiction to show that

$$\sin\theta + \cos\theta \leqslant \sqrt{2}$$

for all values of θ.

Assume that there is a value of θ for which $\sin\theta + \cos\theta > \sqrt{2}$.
Then squaring both sides, we have:

[3]

11.

The function f has domain $(-\infty, 12]$ and is defined by

$$f(x) = e^{4-\frac{x}{3}} + 8.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

12.

The function f has domain $[7, \infty)$ and is defined by

$$f(x) = 1 + \frac{2}{\sqrt{3x-5}}.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

13.

(a) Find $\int x^4 \ln 2x \, dx$. [4]

(b) Use the substitution $u = 10\cos x - 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \sqrt{(10\cos x - 1)}\sin x \, dx.$$ [4]

14. The value £V of a long term investment may be modelled as a continuous variable.
At time t years, the rate of increase of V is directly proportional to the value of V.

(a) Write down a differential equation satisfied by V. [1]

(b) Show that $V = Ae^{kt}$, where A and k are constants. [3]

(c) The value of the investment after 2 years is £292 and its value after 28 years is £637.

(i) Show that $k = 0{\cdot}03$, correct to two decimal places.

(ii) Find the value of A correct to the nearest integer.

(iii) Find the initial value of the investment. Give your answer correct to the nearest pound. [6]

15. (a) Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(i) $\dfrac{1}{\sqrt[4]{9-4x^5}}$ (ii) $\dfrac{3+2x^3}{7-x^3}$ [5]

16. The curve C is defined by

$$3x^3 - 5xy^2 + 2y^4 = 15.$$

The point P has coordinates (1, 2) and lies on C.
Find the equation of the **normal** to C at P. [5]

17. (b) (i) Express $\sqrt{5}\cos\phi + \sqrt{11}\sin\phi$ in the form $R\cos(\phi - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Use your result to part (i) to find the least value of

$$\frac{1}{\sqrt{5}\cos\phi + \sqrt{11}\sin\phi + 6}.$$

Write down a value for ϕ for which this least value occurs. [6]

18. Find a small positive value of x which is an approximate solution of the equation.

$$\cos x - 4\sin x = x^2.$$ [4]

19. Air is pumped into a spherical balloon at the rate of 250 cm^3 per second. When the radius of the balloon is 15 cm, calculate the rate at which the radius is increasing, giving your answer to three decimal places [3]

TOTAL 114